Human Capital and Economic Growth:
Dynamic Implications of Insider-outsider Problem for Macroeconomics*

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Abstract

This paper considers a dynamic model with human capital accumulation, for which both firm-specific skills and general skills are sources of growth. We analyze how the existence of firm-specific skills changes the effects of productivity shocks on economic growth. It is well known that the insider-outsider problem can cause employment inertia in the macro economy because workers with firm-specific skills (“insiders”) face the “hold up problem.” However, most previous studies have been static in nature, so that they have paid little attention to dynamic interactions between firm-specific skills and general skills during the adjustment to the new steady state. This paper considers dynamic models that involve creation of human capital from both firm-specific skills and general skills. We show that the insider-outsider problem that is generated through the creation of firm-specific skills can cause a dramatic decline in the youth labor force during a transition path to the steady state. We also show that the problem may result in a temporary economic downturn even if the shock is positive. In Japan, since the mid-1990s, there has been a dramatic increase in the unemployment rate and a substantial decrease in the working population ratio together with increased irregular

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employment among young people. By analyzing firm-specific human capital as an engine of economic growth, this paper shows that these trends are consistent with our dynamic model. It also demonstrates that the productivity shocks might explain recent dramatic declines in youth employment and temporary declines in growth rates.

Key words: firm-specific human capital, labor markets, insider-outsider problem
JEL classification: E24, J2, J4, O53

I. Introduction

“Life-long labor” contracts, a seniority wage system, and firm-specific human capital investment constitute distinctive, but mutually reinforcing, characteristics of the Japanese labor market.¹ The purpose of this research is to explore the extent to which particularities of the functioning of Japanese labor markets could explain macroeconomic adjustment, increasing unemployment for specific sub-populations, as well as low productivity growth in Japan, since the 1990s. A central issue to be highlighted is how firm-specific human capital formation defines both short-run and long-run labor market adjustment processes and associated macroeconomic performance. More specifically, such firm-specific human capital tends to undermine the functioning of external labor markets, and hence the ability of firms to adapt to macroeconomic shocks. There is an associated insider-outsider problem, which, ceteris paribus, is larger than those that occur when human capital is more generic and worker-specific.

In Japan, the most prominent factor affecting the labor market over the past decade has been the stagnation in growth of productivity and output. This has led to the prolonged stagnation in labor productivity growth, accompanied by a doubling of the unemployment rate over the decade (Figure 1). Many observers agree that the collapse of the asset price bubble of the late 1980s and mistakes in macro and micro policy have been the primary causes for why the slump has been prolonged. There are, however, a number of other explanations. In particular, the question remains as to whether the Japanese economic system, including employment practices, has also been responsible.²

In recent literature, Hayashi and Prescott (2002) document that treating TFP (total factor productivity) as exogenous, growth theory accounts well for the Japanese lost decade of growth. They propose that the decline of TFP, as well as a dramatic decline in labor hours, was the source of the stagnation in output growth. Other recent studies, however, report that a permanent productivity slowdown was moderate in Japan in the 1990s. In the following

¹ See, for example, Mincer and Higuchi (1988), Hashimoto (1990, 1995) and Ito (1992).
² See, for example, Genda and Rebick (2000) for an overview of Japanese labor in the 1990s.
analysis, we explore how firm-specific human capital and the resulting hold-up problem magnify the negative impacts of the mild productivity slowdown.

In the model, there are workers whose skills are either firm-specific or general. Workers obtain firm-specific skills if and only if they keep working at the same firm. Turnover costs and firm-specific human capital generally drive a wedge between the lowest wage for which a skilled employee will work and the highest wage the employer will pay. They thus generate a rent to continued employment as well as a seniority wage. If the shocks were temporary, Japanese firms would have minimized the costs of the hold-up problem through adjusting hours of work and inventory. However, when the productivity shocks are persistent, the hold-up problem can cause various losses in Japanese labor markets even under the mild shocks. In the short-run, the losses tend to arise as overemployment of workers with firm-specific skills and underemployment of workers with general skills. In particular, under endogenous labor supply, unemployment prevails only among young workers. In the 1990s, there were salient empirical observations that young workers dramatically increased unemployment rates and reduced labor participation ratios. Our theoretical model implies that the observations during the past decade can be a bad symptom in predicting another prolonged stagnation in Japan.

In his seminal work, Becker (1975) argues that the firm will not be willing to invest in general training when labor markets are competitive but is willing to invest in specific training because it cannot be transferred to outside the firm. The provision of firm-specific training, however, causes a hold-up problem. In Japanese labor markets, the problem seems more
serious under productivity slowdown because of their large amount of firm-specific human capital investment. The hold-up problem seems to show potential promise for understanding aspects of labor markets in Japan.

There are a large number of studies that investigate the hold-up problem in labor markets (see, for example, Malcomson [1997] for their overview). The hold-up literature is concerned with economic relationships with the following characteristics: (1) because of turnover costs or specific investments, there are rents to continuing a relationship once started that are, in principle, available for the parties to bargain over; and (2) there are problems in writing contracts contingent on all the future events that are important for the relationship. In this paper, we discuss these characteristics in labor markets from a macroeconomic perspective. We analyze how the existence of firm-specific skills changes the effects of productivity shocks on economic growth.

It is well known that the insider-outsider problem can cause employment inertia in the macro economy because workers with firm-specific skills (“insiders”) face the “hold up problem” (see, for example, Blanchard and Summers [1990]). However, except for Fukao and Otaki (1993), most previous studies have been static in nature, so that they have paid little attention to dynamic interactions between firm-specific skills and general skills during the adjustment to the new steady state. This paper considers dynamic models that involve creation of human capital from both firm-specific skills and general skills. We show that the insider-outsider problem that is generated through the creation of firm-specific skills can cause a dramatic decline in the youth labor force during a transition path to the steady state. We also show that the problem may result in a temporary economic downturn even if the shock is positive.

In our model, the prolonged stagnations are initiated by exogenous external shocks. This paper thus cannot explain why the slump started in Japan in the 1990s. It, however, verifies why the slump has been so prolonged even if the exogenous shocks were moderate. Until the late 1980s, mutually reinforcing characteristics of the Japanese labor market had served Japan well. They, however, started magnifying the negative impacts under a persistent productivity slowdown. The economic system which had served well during previous decades was then due for a rapid transformation.

The rest of this paper is organized in the following way: After overviewsing salient characteristics of Japanese labor market, Section III proposes a simplified two-period framework in order to examine the implications of an unanticipated demand shock, when a firm makes an intertemporal investment in firm-specific human capital. Section IV–V then extend this basic analysis to consider a multi-period overlapping-generations version of the model, which also allows for consumers’ optimal labor-leisure decisions affected rate of labor force participation.
II. Salient Characteristics of Japanese Labor Market and Macroeconomic Performance in the 1990s and the 2000s

When we look at various indicators in Japanese labor markets in the 1990s and the 2000s, we can see salient empirical observations that indicate a dramatic structural change in the labor markets. The change occurred mainly for young workers (under the age of 25). The following four are among the salient empirical observations:

1. The sharp increase in unemployment, which is particularly acute for young male workers. (Figure 2)

Unemployment remained at low levels by international standards until the mid 1990s, when the rate climbed abruptly from 3 percent, exceeding 5 percent in the summer of 1999. This was particularly pronounced in the case of young men (under the age of 25) as well as older men (age 60–64), both having rates over 10 percent. There has been an increase in the proportion of long-term unemployed with out of work spell over one year, and the proportion of long-term unemployment among prime-aged workers between the ages of 30 and 50 now exceeds 20 percent.
2. Average labor participation ratios of male workers were almost stable over the 1990s and the 2000s. There was, however, a dramatic downturn in labor participation for young male workers. (Figure 3)

Although unemployment rates have risen, employment rates have not fallen. Average male participation rates have been stable over the 1990s and the 2000s under the two-point rise in the unemployment rate. There was, however, a dramatic decline in labor participation of young men (under the age of 25) over the 1990s. Labor participation ratios of older men (age 60–64), in contrast, showed only a marginal decline over the 1990s and the 2000s.

3. The consequence of the foregoing was a dramatic fall in the number of male regular workers for specific young age groups. (Figure 4)

In addition to a dramatic decline in labor participation of young men, there was a substantial decline in the number of young male regular workers under the age of 20 over the 1990s. The number of young male regular workers under the age of 25 also started to show a substantial decline in the 2000s. The number of elder regular workers was, in contrast, stable over the 1990s and the 2000s.

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**Figure 3. Labor Participation Ratios (Male, 1990=1)**

- **All ages**
- **15~19**
- **20~24**
- **60~64**
4. On the other hand, there was a large and sustained upswing in labor costs, as a share of total value added. (Figure 5)

There was a large and sustained upswing in labor costs, as a share of total value added. Much of the increase in the share of labor costs can be attributed to a sharp increase in wages from the late 80s through the mid-90s.

The above salient empirical observations indicate that a dramatic structural change in Japanese labor markets in the 1990s occurred mainly for young workers (under the age of 25). In the following sections, we investigate how the hold-up problem can explain these empirical observations. We also explore whether high unemployment rates and low labor participation of young workers that were observed during the past decade can be a bad symptom for another prolonged stagnation in Japan.
III. Labor Market Adjustments with Firm-Specific Human Capital: A Descriptive Explanation

III.1. The Ex Ante Analysis

In macroeconomics, there are a large number of studies that investigated hold-up problems in labor markets in the context of the insider-outsider problem. The purpose of this section is to provide a descriptive and intuitive explanation for the insider-outsider problem in our labor market. Let us consider a single representative firm that may hire two types of workers, insiders and outsiders. The two types of workers are distinguished only by their firm-specific human capital. To the extent that they keep working at the same firm, insiders have higher human capital than outsiders because of learning-by-doing and/or firm-specific trainings in previous periods. However, when they quit the firm, insiders lose all of firm-specific human capital. Consequently, there is no difference in human capital between two types of workers when working at a new firm. Both types of workers always have an outside job opportunity for which they can earn constant wage, $w_t^{us}$.

Figure 6 shows labor supply curves and marginal product curves of insiders for the representative firm. For simplicity, we assume that labor hour is fixed for each worker and that the number of insiders is equal to $N_t^s$ at the beginning of the period. Then, because of the outside job opportunity, the firm faces a flat supply labor curve at $w_t = w_t^{us}$. However, because of the total number of insiders is fixed, the labor supply curve becomes vertical at $N_t = N_t^s$ for insiders.
In Figure 6, two downward sloping curves, \( \text{MPL}^U \) and \( \text{MPL}^S \), show marginal product of labor of insiders with and without firm-specific human capital. Because of firm-specific human capital, \( \text{MPL}^S \) is always higher than \( \text{MPL}^U \) for the firm. The firm’s demands for outsiders and insiders are obtained by maximizing the profit. However, unlike standard competitive labor markets, neither \( \text{MPL}^U \) nor \( \text{MPL}^S \) is labor demand curve for the workers. This is because the firm can hire insiders at the wage rate that is lower than their marginal product of labor.

The wage of insiders, \( w^* \), is determined by a bargaining game between the workers and the firm. Suppose that \( \text{MPL}^S = w^* \) at \( N_t = N_t^S \) for insiders. Then, since hiring outsiders is more profitable for the firm when \( w^* > w^* \), the threat point of the firm is \( w^* \). In contrast, because of the outside job opportunity, the treat point of the insiders is \( w^{us} \). Therefore, Nash bargaining
solution suggests that the firm hires all of the insiders at the wage

\[ w_s = \gamma w^* + (1 - \gamma) w^{us}, \quad (0 < \gamma < 1) \]

where \( \gamma \) is a parameter that denotes the bargaining power of the insiders.

To the extent that the firm can hire insiders at the wage \( w_s \) that is lower than \( w^* \), the firm has no incentive to hire outsiders. The Nash bargaining solution (1) suggests that there is no demand for outsiders unless the firm faces unexpected positive shocks after the bargaining.

III.2. The Ex Post Analysis

Let us now assume that after the insiders and the firm reached the agreement at the bargaining, there is an unexpected structural change entailing a substantial fall in demand for the homogeneous product. The structural change shifts both \( MPL^U \) and \( MPL^S \) downward. However, if the new gross marginal product of insiders remains above the wage rate, \( w^S \), both the demand and supply for insiders is perfectly inelastic in the short run. A crucial insight is that because of the firm-specific human capital investment there is a hold-up problem for both the employer and employees. For the firm, the existing expenditures on training workers are a sunk cost. Given this bygone, the ex post incentive for the firm is to keep its insiders, provided that the value of their gross marginal product is greater than the ex post wage rate, which could fall depending on the relative bargaining power of the firm and its skilled workers. For the insiders, the hold-up problem may even be stronger, since to the extent their human capital is firm specific, the outside wage that they face is the same as for outsiders. Furthermore, that outside wage rate has been depressed by the negative labor demand shock for outsiders.

The interaction between ex post marginal product of labor as well as labor supply curve is presented in Figure 7. The foregoing analysis of the insider-outsider problem with firm-specific human capital formation is analytical akin to the well-known “hold up” problem in the theory of bargaining between two agents. In this case, one party principally bears the costs of investment, for which the value is of specific use to the other party. The hold-up problem offers a rationale for vertical integration in the face of a specific investment and bilateral bargaining. It entails in effect a form of vertical integration with the labor market, in order that a firm may internalize the benefits from specific investments, which constitute a mutually beneficial exchange between itself and workers. However, the unanticipated shock generates an ex post sub-optimality in this human capital investment for the investing firm. If it had anticipated such a “bust” scenario, the firm would have undertaken less human capital investment and hired fewer “skilled” workers.

There is also an apparent relation between the hold-up effect of firm-specific human capital investment and insider-outsider models of unemployment. In the latter, it is a labor union,
which leads to a wage gap that favors workers within the firm, who receive more than the marginal value product of their labor input. In contrast, to these protected insiders, equally productive outsiders are disadvantaged in the external labor market. In the case of firm-specific human capital, the skilled workers may receive wages (depending on their bargaining power), which are equal to their net marginal value product within the firm. However, those wages are higher than the outside wage they face on the labor market and the potential external marginal value product of their labor supply.

**Figure 7. Labor Market Equilibrium After the Shock**
IV. An Overlapping Generations Model with Firm-Specific Human Capital: A Case of Two Periods

IV.1. A Representative Firm

In the following analysis, we consider overlapping-generations models with human capital accumulation. For simplicity, this section first considers a model where individuals live for two periods: young and old. They supply labor during both periods. They are genetically identical within and across generations. Some old workers, however, have firm-specific skills if and only if they work at the same firm for two consecutive periods. The other old workers have general skills if and only if they had education when they were young.

We consider a competitive world where economic activity is performed over infinite discrete time. There is a single perishable final good, taken as a numeraire; it is competitively produced. The firm can hire four types of workers: young workers $N_{0,t}$, unskilled old workers $N_{us,t}$, old workers with firm-specific skills $N_{s,t}$, and old workers with general skills $N_{h,t}$. Old workers with either firm-specific or general skills have higher quality than unskilled old workers in the sense that their effective labor supply is larger. Denoting output in period $t$ by $Y_t$, the production function in period $t$ is specified as

$$
(2) \quad Y_t = A_t f(N_{0,t}, q N_{s,1,t} + N_{us,1,t} + e N_{h,1,t}),
$$

where $q$ is the quality of firm-specific skills and $e$ is the quality of general skills. The function $f$ satisfies that $f_1 > 0$, $f_2 > 0$, $f_{11} < 0$, and $f_{22} < 0$. Under the production function (2), the marginal products of old workers are equalized among all types of workers once their quality is adjusted. We assume that $e > q > 1$ in the following analysis.

The profit in period $t$ is equal to output in period $t$ minus the costs of production in period $t$. The costs of production are the sum of several types of labor costs. Denote the wages for young workers, unskilled old workers, and old workers with firm-specific skills by $w_{0,t}$, $w_{us,t}$, and $w_{s,t}$ respectively. Then, since the wage for old workers with general skills is determined by $e w_{us,t}$ under the production function (2), the profit in period $t$ is written as

$$
(3) \quad \pi_t = A_t f(N_{0,t}, q N_{s,1,t} + N_{us,1,t} + e N_{h,1,t}) - w_{0,t} N_{0,t} - w_{s,t} N_{s,1,t} - w_{us,t} N_{us,1,t} - e w_{us,t} N_{h,1,t},
$$

In each period, the firm maximizes the present value of current and future profits as follows

$$
(4) \quad \text{Max } \Pi_t = \sum_{i=0}^{\infty} \delta^i \pi_{t+i}
$$
where $\delta$ is discount factor such that $0 < \delta < 1$.

Under the profit maximization, the wages for unskilled old workers and old workers with general skills are determined by the marginal products of labor. The wage for old workers with firm-specific skills, $w_{st}$, is, however, determined by a bargaining game between the firm and the skilled workers. This is because the old workers face a hold-up problem when their skills are firm-specific. The old workers with firm-specific skills receive only the competitive wage of unskilled old workers when they quit the firm, while their marginal products of labor in period $t$ is equal to $q w_{us}^t$. Like equation (1), the bargaining game thus leads to

$$
(1)' \quad w_{st}^t = [\gamma + (1-\gamma) q] w_{us}^t, \quad \text{where } 0 < \gamma < 1.
$$

where $\gamma$ denotes the bargaining power of the firm that lies between zero and one. One may argue that the hold-up problem becomes more serious for the old workers as $\gamma$ increases.

Since $\gamma$ is less than one, $w_{st}$ is smaller than the marginal product of the old workers with firm-specific skills $\partial Y_t/\partial N_s^t$. The firm thus always hires all of the existing old workers with firm-specific skills. On the other hand, to the extent that $\gamma > 0$, the old workers with firm-specific skills never have an incentive to quit the firm. This implies that the number of the old workers with firm-specific skills is equal to the number of the young workers in the previous period, that is,

$$
(5) \quad N_{1,t}^s = N_{0,t-1}^s \text{ and } N_{us,1,t}^s = 0.
$$

As in the last section, the conditions hold even if there are some unexpected exogenous shocks in period $t$.

In period $t$, the firm decides the number of young workers $N_{0,t}$ so as to maximize the present value of profits under the constraint that $N_{1,t+1}^s = N_{0,t}$. The first-order conditions $\partial \Pi_t/\partial N_{0,t} = 0$ and $\partial \Pi_t/\partial N_{1,t}^h = 0$ thus lead to

$$
(6) \quad A_t f_1(N_{0,t}^h, q N_{0,t-1}^h + c N_{1,t}^h) + \delta q A_{t+1} f_2(N_{0,t+1}^h, q N_{0,t}^h + c N_{1,t+1}^h) = w_{0,t}^h + \delta w_{st}^t,
$$

$$
(7) \quad A_t f_2(N_{0,t}^h, q N_{0,t-1}^h + c N_{1,t}^h) = w_{us}^t.
$$

The wage for the young workers, $w_{0,t}$, is determined by their reservation wage. Under the reservation wage, the young is indifferent between employed and unemployed. If and if only unemployed young workers take education, they will have general skills when old. For simplicity, we assume that there is neither education cost nor disutility of labor. The arbitrage condition for the young workers is written as $w_{0,t} + \delta w_{st}^t = \delta c w_{us}^t$. We therefore obtain

$$
(8) \quad w_{0,t} = \delta [e - \gamma - (1-\gamma) q] w_{us}^t,
$$
The condition (8) states that the equilibrium wage for the young workers is decreasing in the quality of firm-specific human capital $q$. This reflects the fact that the gap between $w^{st}_t$ and $w^{us}_t$ increases as $q$ increases. Larger firm-specific skills are, more likely the young accept lower wage. Equation (8) implies that the young workers may receive the wages that are less than the marginal products of labor. This is because the young can upgrade to skilled workers in the next period when they work for the firm.

Without disutility of labor, all old workers always supply the fixed amount of labor. The equilibrium condition for the old workers is thus described by

$$N^{b}_{1,t} + N^{h}_{1,t} = N^*.$$  

where $N^*$ is the total number of population of each generation.

In the following analysis, we denote the growth rate of technology by $a = A_{t+1}/A_t$ and assume that it is constant over time. Substituting (1)', (8), and (9) into (6) and (7), we obtain

$$f_1(N_{0,t-1}, eN^* + (q-e)N_{0,t-1}) = a \delta (e-q) f_2(N_{0,t-1}, eN^* + (q-e)N_{0,t-1}),$$

$$A_t f_2(N_{0,t}, eN^* + (q-e)N_{0,t-1}) = w^{us}_t.$$  

Equation (10) denotes the dynamics of $N_{0,t}$ that shows how the number of the young workers evolves over time. Given $N_{0,t-1}$ and $N_{0,t}$, equation (11) determines the equilibrium value of $w^{us}_t$. Once $w^{us}_t$ is fixed, equations (1)' and (8) determine the equilibrium value of $w^{0,t}_t$ and $w^{s}_t$ respectively.

### IV.2. The Steady State

The goal of our theoretical analysis is to explore what impacts various productivity shocks have on employment in our model. Specifically, we investigate what impacts permanent declines of three productivity parameters $a$, $q$, and $e$ will have on employment and total production in the economy. The first-round impact of the productivity shock is the changes of the marginal product of labor. However, to the extent that the change of productivity is small, the insider–outsider problem suggests that the condition (5) remains true before and after the shock. That is, the firm keeps hiring all of the existing old workers with firm-specific skills. This causes an overshooting of the number of young workers in the short-run. The impacts, thus, depend on what time span we are interested in. This section first explores the impacts on the steady state.

In the steady state, the number of young workers $N_{0,t}$ remains constant for all $t$. Denoting the constant value by $N_0$, equation (10) leads to
In other words, the steady state value of $N_0, \epsilon$ is determined by equation (12). Taking total differentiation of (12) with respect to $a, q$, and $\epsilon$, we obtain

$$
\frac{\Delta N_0}{\Delta a} = \frac{-\delta(e-\delta)f_2}{(1+a\delta)(e-q)f_{12} - f_{11} - a\delta(e-q)^2 f_{22}},
$$

$$
\frac{\Delta N_0}{\Delta q} = \frac{a\delta f_2 + \{f_{12} - a\delta(e-q)f_{22}\}N_0}{(1+a\delta)(e-q)f_{12} - f_{11} - a\delta(e-q)^2 f_{22}}.
$$

$$
\frac{\Delta N_0}{\Delta \epsilon} = \frac{-a\delta f_2 + \{f_{12} - a\delta(e-q)f_{22}\}(N^*-N_0)}{(1+a\delta)(e-q)f_{12} - f_{11} - a\delta(e-q)^2 f_{22}}.
$$

Since $f_{11} < 0, f_{22} < 0$, and $f_2 > 0$, equation (14) implies that if $f_{12} \geq 0$, an increase in $q$ always raises $N_0$. This is mainly because an improvement of firm-specific skills makes working in the young more attractive. However, if $f_{12} < 0$, we can no longer pin down the sign of equation (14) in general. The right-hand side of equation (14) depends on various parameters, which suggests that the total impacts of the productivity shock $q$ on $N_0$ also depend on various other factors.

More interestingly, we can show that the sign may not be positive for equations (13) and (15) even if $f_{12} \geq 0$. In case of equation (13), the sign becomes negative if $f_{12} \geq 0$. This implies that a rise of $a$ reduces the employment of the young workers. This happens because an improvement of total factor productivity growth implies higher marginal productivity in the future, so that the young prefers education for general skills to working for firm-specific skills when $\epsilon > q$. In contrast, the sign of equation (15) is ambiguous when $f_{12} \geq 0$. In the right-hand side of (15), the denominator is always positive if $f_{12} \geq 0$. But the first-term of the numerator is negative, while the second-term is positive if $f_{12} \geq 0$. An improvement in education quality, $\epsilon$, does not have a definitive impact on the employment of the young workers in our model. This happens because an improvement of education quality not only makes education in the young more attractive through a substitution effect but also increases an opportunity cost of education in the young through increasing the wage of the young workers. The latter effect may dominate the former when $N^*-N_0$ is large in (15). The substitution effect becomes small when the young workers are scarce.
IV.3. The dynamic equilibrium

In our overlapping generations model, the equilibrium value of $N_{0,t}$ has a saddle-point path that converges to its steady state under some conditions. However, we cannot derive an explicit form of the dynamic equation without specifying the production function. In the following analysis, we assume that the production function (2) takes the form of the Cobb-Douglass production function as follows:

\[ Y_t = A_t (N_{0,t}^q N_{1,t}^u + e N_{1,t}^s)^{1-\eta}, \quad \text{where } 0 < \eta < 1. \]

Then, equation (10) implies that the equilibrium value of $N_{0,t}$ satisfies

\[ \eta[(e N^* + (q-e) N_{0,t-1})/N_{0,t}]^{1-\eta} = a \delta (1-\eta) (e-q) [(e N^* + (q-e) N_{0,t})/N_{0,t+1}]^{-\eta}. \]

The steady state of $N_{0,t}$ is

\[ N_0 = \eta e N^*/[(e-q)[a \delta (1-\eta) \eta]]. \]

In the following analysis, we assume that $e/q > 1 + (1-a \delta) \eta/a \delta (1-\eta)$, so that $0 < N_0 < N^*$. The dynamic equation then converges to the steady state if and only if $0 < \eta < \min[1/2, a \delta/(1+a \delta)]$. The dynamic equation is written as $(e N^* + (q-e) N_{0,t-1})/N_{0,t} = a \delta (e-q) (1-\eta)/\eta$, or equivalently,

\[ N_{0,t} = B - C N_{0,t-1}, \]

where $B = \eta e N^*/[a \delta (e-q) (1-\eta)]$ and $C = \eta/[a \delta (1-\eta)]$. Since $0 < C < 1$ when $0 < \eta < a \delta/(1+a \delta)$, equation (19) implies that the equilibrium value of $N_{0,t}$ converges to the steady state with fluctuations around the steady state.

V. An Extension to a $n+1$ Generations Model

V.1. The Model

In the last section, we explored an overlapping generations model when individuals live for two periods. The two period model is useful to understand essential features of the model. However, most of individuals usually work for more than 40 years in our world. Consequently, the number of workers that have some work experience is usually larger than the number of workers that have no work experience in the economy. It is therefore more realistic to consider an overlapping generations model when individuals live for longer periods.
In this section, we extend our overlapping generations model to the case where individuals live for \( n+1 \) periods. As in the last section, we consider four types of workers: young workers \( N_{0,t} \), unskilled old workers, old workers with firm-specific skills, and old workers with general skills. For simplicity, the workers are young only for one period and they work as old workers for the rest of the periods. Denote unskilled old workers by \( N_{us,j,t} \), old workers with firm-specific skills by \( N_{s,j,t} \), and old workers with general skills by \( N_{h,j,t} \) when their age is \( j+1 \) in period \( t \). The period \( t \) production function is then written as

\[
Y_t = A_t f \left[ N_{0,t}, \sum_{j=1}^{n} \left( q N_{s,j,t} + N_{us,j,t} + e N_{h,j,t} \right) \right]
\]

The production function implies that once their quality is adjusted, the marginal products of all types of old workers are equalized regardless of their ages. We assume that the old workers have firm-specific skills if and only if they had education when they were young, while they have general skills if and only if they had education when they were young. We also assume that the young are unemployed when receiving education.

Without disutility of labor and population growth, the total labor supply of age \( j+1 \) old workers is \( N^* \) for all \( j = 1, 2, ..., n \). As in the last section, the wages are determined by equations (4) and (8). Since the old workers with firm-specific skills never have an incentive to quit the firm, it holds that \( N_{s,j,t} = N_{0,t-j} \) and \( N_{us,j,t} = 0 \) for all \( j = 1, 2, ..., n \). The first-order condition thus leads to the following dynamic equation:

\[
f_1 \left[ N_{0,t}, \sum_{j=1}^{n} \left( eN^* + (q - e)N_{0,t-j} \right) \right] = (e-q) \sum_{k=1}^{n} (a\delta)^k f_2 \left[ N_{0,t+k}, \sum_{j=1}^{n} (eN^* + (q - e)N_{0,t+k-j}) \right].
\]

Equation (21) implies that the steady state equilibrium \( N_0 \) is determined by

\[
f_1[N_0, n (e N^* + (q - e) N_0)] = [a\delta/(1 - a\delta)] (e-q) f_2[N_0, n (e N^* + (q - e) N_0)].
\]

Needless to say, equation (22) is degenerated into equation (12) when \( n = 1 \).

V.2. Dynamics

An explicit form of the dynamic equation can be obtained specifying the production function. Assuming that the production function takes the Cobb–Douglass form, the period \( t \) production function is written as follows
where \( 0 < \eta < 1 \). Under the production function, (21) is written as

\[
(24) \quad \eta \left[ \sum_{j=1}^{n} \left( eN^* + (q - e)N_{0,t-j} \right) / N_{0,t} \right]^{1-\eta} = (e-q) \left( 1-\eta \right) \sum_{k=1}^{n} (a\delta)^k \left[ \sum_{j=1}^{n} \left( eN^* + (q - e)N_{0,t+j+k} \right) / N_{0,t+k} \right]^{-\eta},
\]

The steady state is explicitly solved as

\[
(25) \quad N_0 = (1-a\delta) \eta e nN^*/[(e-q)[a\delta(1-\eta)(1-a\delta)\eta]].
\]

In the following analysis, we assume that \( a\delta < 1 \) and \( e/q > 1 \). Under a weak condition, the dynamic equation has a saddle-point path that converges to the steady state.

The equation is written as

\[
\sum_{j=1}^{n} \left( eN^* + (q - e)N_{0,t-j} \right) / N_{0,t} = [a\delta/(1-a\delta)] (e-q) (1-\eta)/\eta, \quad \text{or equivalently,}
\]

\[
(26) \quad N_{0,t} = F - G \sum_{j=1}^{n} N_{0,t-j},
\]

where \( F = (1-a\delta) \eta e nN^*/[a\delta (e-q) (1-\eta)] \) and \( G = (1-a\delta) \eta /[a\delta (1-\eta)] \).

V.3. Simulation

We now consider how the number of young workers, \( N_{0,t} \), and the growth rate of output, \( \Delta Y_t/Y_t \), will change when there are unexpected changes in the productivity in period 1. In the experiment, we normalize population of each generation to be one, that is, \( N^* = 1 \). We set \( a = 1.1, \delta = 0.8, \eta = 0.2, e = 1.6, q = 1.2, \) and \( n = 8 \) as benchmark parameters and assume that they remained unchanged until period 0. We then investigate how \( N_{0,t} \) and \( \Delta Y_t/Y_t \) will change when there was one of the following three events; (i) The parameter “a” unexpectedly increases from 1.1 to 1.155 in period 1, so that total factor productivity growth increases permanently by 5% after period 1. (ii) The parameter “q” unexpectedly declines from 1.2 to 1.14 in period 1, so that the quality of firm-specific skills decreases permanently by 5% after period 1. (iii) The parameter “e” unexpectedly increases from 1.6 to 1.68 in period 1, so that the quality of general skills
increases permanently by 5% after period 1. For simplicity, we assume that all endogenous variables are at the steady state before period 1. Among the above three events, (i) and (iii) are positive productivity shocks, while (ii) is a negative productivity shock. Since we set $n = 8$, we can interpret that one period is about five years in the following simulations.

Figures 8–(i) and 8–(ii) show dynamic paths of the number of the young workers $N_0, t$ and the growth rate of total output $\Delta Y_t/Y_t$ when the parameter “$a$” increases by 5%. In the figures, the number of young workers shows significant declines. After showing up and down convergence to the steady state, it declines by more than 30 points in the long–run (Figure 8(i)). The improvement of total factor productivity growth reduces the employment of the young workers in the long–run because the young prefer education (training?) for general skills. What is noteworthy is that the number of young workers declines more in the short–run. In our model, the decline of the young workers is larger in the short–run because workers with firm–specific skills will not quit the firm until their retirement. The model therefore has overemployment of workers with firm–specific skills and underemployment of young workers in the short–run. The decline of employment would have been diversified to skilled workers if their demand for labor was determined by their marginal products of labor. Such diversification is not profitable for the firm because of the hold–up problem.

![Figure 8-(i). A Rise of a and Employment Dynamics of the Young Workers](image-url)
The rise of $a$ increases the growth rate of output by more than 2 points in the long-run. This is a natural consequence of a positive permanent shock on the total factor productivity growth. The growth rates are even higher from period 2 to period 8. However, the positive impacts are dominated by the declines in the number of young workers in period 1. The permanent increase of the total factor productivity growth leads to a temporary recession because of the decline of labor input. After the temporary drop, the growth rate of output is boosted up before gradual convergence to the steady state (Figure 8(ii)).

Figures 9-(i) and 9-(ii) show dynamic paths of the number of the young workers $N_{0,t}$ and the growth rate of total output $\Delta Y_t / Y_t$ when the parameter "q" decreases by 5%. In the figures, the number of young workers shows significant declines. After showing up and down convergence to the steady state, it declines by more than 13 points in the long-run (Figure 9(i)). The decrease of $q$, which is a deterioration of firm-specific skills, reduces the employment of the young workers because the young prefers education for general skills. The reduction is more dramatic in the short-run. The impacts on the employment of the young and old workers change as the time passes. This is because the decline of young workers reduces the number of workers with firm-specific skills in the following periods. Since $N^s_{T+1} = N_{0,T}$, the number of workers with firm-specific skills definitively starts to decline after period 2. However, the decline of the workers with firm-specific skills would be moderate because they will not quit until the retirement.
The fall of $q$ does not have a significant impact on the growth rate of output in the long-run. This is because the negative impacts are eventually substituted by positive impacts by increased old workers with general skills. However, since it takes time to increase the number of old workers with general skills, the long-run impacts are dominated by declines in the number of effective workers in the short-run. The growth rate of output declines by 1.5 points in period 1 before gradual convergence to the steady state (Figure 9(ii)).
Figures 10–(i) and 10–(ii) show dynamic paths of the number of the young workers $N_{0,t}$ and the growth rate of total output $\Delta Y_t/Y_t$ when the parameter “$e$” increases by 5%. In the figures, the number of young workers shows significant declines. After showing up and down convergence to the steady state, it declines by about 12.5 points in the long-run (Figure 10(i)). The increase of $e$, which is an improvement of general skills, reduces the employment of the young workers in the long-run because the young prefer education (training?) for general skills. However, the reduction is larger in the short-run because workers with firm-specific skills will not quit the firm until the retirement. As in the above two cases, the model has
over-employment of workers with firm-specific skills and under-employment of young workers temporarily.

The rise of $\epsilon$ does not have a significant impact on the growth rate of output in the long-run. The positive impacts are cancelled out because old workers with general skills decline at the same time. In particular, the long-run positive impacts are dominated by the declines in the number of young workers in the short-run. The growth rate of output declines temporarily in period 1 before gradual convergence to the steady state (Figure 10(ii)).

VI. Implications for Japanese Economy

In the 1990s and the early 2000s, the Japanese economy experienced a prolonged decline of economic growth that was accompanied by a dramatic structural change in the labor market. The structural change was especially conspicuous for unemployment rates and labor participation rates among young workers. In contrast, an increase of unemployment rates and a decline of labor participation rates were moderate among middle-age workers. The number of regular workers decline substantially among young workers, while regular workers have been still dominant among workers over 30 years old. If we interpret regular workers as workers with firm-specific skills, the simulation results presented in the last section are consistent with these stylized facts in recent Japanese labor market.

In Figures 8, 9, and 10, the productivity shocks caused a temporal decline of economic growth accompanied by a drastic decline of young workers’ employment. Consequently, the share of older workers’ employment increased under the recession. This happened partly because the older workers have less incentive to be unemployed than the young workers in our model. However, what is more noteworthy in our model is that insiders who have firm-specific skills remained employed by the same firm because of a hold-up problem.

For example, Figure 11 depicts how the share of older workers’ employment will change when the parameter $q$ declines by 5 points. The share of older workers’ employment increases by 1.15 points even in the long-run. But it increases more dramatically in the short-run. Soon after the shock, it increased by about 1.47 points. This reflects a temporal over-employment of the older workers under our insider-outsider problem.
Under globalization, general skills are becoming more important in many Japanese firms. In contrast, under the dramatic structural change, the role of firm-specific skills became less important in improving productivity. A rise of $e$ and a decline of $q$ are the productivity shocks that Japanese economy has experienced during the last two decades. The structural changes in the labor markets will have several beneficial effects in the long-run. However, our models suggest that they might be accompanied by overemployment of old regular workers and under-employment of young workers in the short-run.

It is worthwhile to note that “the short-run” in the model is not necessarily short in years. In our world, most of individuals usually work for more than 40 years. Consequently, the number of workers that have some work experience is usually larger than the number of workers that have no work experience in the economy. In our $n+1$ generations model, we set $n = 8$. In such a model, one period is about five years. This implies that the temporal decline of growth rate and the overemployment of old regular workers may persist for about 5 years until the retirement age even if there are permanent structural changes. A hold-up problem may cause some misallocations of labor force in the economy.

**VII. Conclusion**

Many labor markets cannot be adequately described as spot markets. Turnover costs and firm-specific investments drive a wedge between the lowest wage for which a skilled employee will work and the highest wage the employer will pay. They thus generate a rent to continued employment of skilled workers that is the source of the hold-up problem in our model.

When the productivity slowdown is persistent, the hold-up problem causes various losses such as high unemployment rates and low labor participation of young workers in Japan even under the mild shock. In the short-run, the losses are reflected mainly in high unemployment.
rates of unskilled young workers. The losses, however, eventually decrease the number of skilled workers and have serious negative impacts on total output in the following periods.

In Japan, since the mid-1990s, there has been a dramatic increase in the unemployment rate and a substantial decrease in the working population ratio together with increased irregular employment among young people. Entering into the 2000s, these trends have even spread to the 20 to 29 age group. On the other hand, rises in the unemployment rate and decreases in the working population rate have been limited in the 30 to 39 and 40 to 49 age groups. These age groups have had extremely large percentages among the regular employment population even in the 2000s. By analyzing firm-specific human capital as an engine of economic growth, these trends are consistent with our dynamic model. It also demonstrates that the productivity shocks might explain recent dramatic declines in youth employment and temporary declines in growth rates in Japan.
References