Corruption, Default and Optimal Credit in Welfare Programs

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Abstract

In this paper we present a dynamic model of subsidized credit provision to examine how asymmetric information exacerbates inefficiency caused by corruption. Though designed to empower the underprivileged, the fate of such credit programs largely depends on the efficiency of the credit delivery system. Corruption often erodes this efficiency. Nevertheless, when a corrupt loan official and a borrower interact with symmetric information, credit terms can be so designed that corruption will affect only the size of the surplus, but not repayment. With private information on the borrower’s productivity this result changes. The corrupt loan official may induce the low productivity borrower to default, mainly because of high revelation costs. The government can improve the repayment rate, but will have to under-provide the first period loan. On the other hand it can permit default by the low productivity borrower, and maintain a higher credit level. The second option may sometimes be preferred. This inefficient outcome is caused by two factors - informational ratchet effects and countervailing incentives, which are commonly present in many agency relationships.

Keywords: Corruption, Information rent, Countervailing incentives, Ratchet effect
JEL Classification: H2;D8;K4

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1 Introduction

Subsidized credit programs are quite common in developing countries. These may include providing cheap loans to farmers, special credit to small businesses, and subsidy to exporters and so on and so forth. But such programs suffer from two problems: high incidence of default (Hoff and Stiglitz; 1990) and corruption in implementation (Rose-Ackerman, 1999). Economists generally focus on the default problem and their explanations vary from adverse selections, imperfect screening of borrowers, and incorrect design of credit contracts to wrong targeting. Political economists, on the other hand, provide a great deal of evidence of corruption, and their concern is often shared by aid agencies and practitioners. Thus, there can be an association between corruption and default that may not be coincidental. Unfortunately, no empirical and theoretical studies have examined the link between the two phenomena. The present paper makes a theoretical attempt to do so with the help of a dynamic model of credit provision.

In general, corruption in credit provision is common to a large number of countries. At a micro level, various field studies in India report two types of corruption in the provision of agricultural credit through government agencies. One type of corruption involves diverting credit to the rich or locally powerful farmers. Such powerful borrowers are also believed to be big defaulters (Sarap, 1991). The other type of corruption involves poor or not-so-powerful

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1Hoff and Stiglitz (1990) note that ‘high default rates have prevented (formal lending) institutions from being self-financing’, and ‘despite these subsidies, many of these credit programs have had very little success in reaching farmers without collateral or with below-average income.’ Thus, Hoff and Stiglitz emphasize on two aspects: high default rates and designs or delivery of credit.

2The World Bank (1997, p.59) noted that in South Asia in 1991-92 only 10 percent of public subsidies reached the households below median income. In a similar vein, India’s former Prime Minister, the late Rajiv Gandhi, once lamented that of every rupee that was spent for the poor, only 20 percent eventually reached the target.

3In Kenya, it was estimated that a third of banking assets in 1992 were rendered worthless because of political interference and favoritism (Rose-Ackerman, 1999, p.10). In rural Pakistan, one field study conducted in 1980-81, found 30 percent of loans from government operated banks were getting defaulted, while the same rate for the local moneylender was only 2.7 percent (Hoff and Stiglitz, 1990).

4Rose-Ackerman (1999, p.10) writes, “If the supply of credit and the rate of interest are controlled by the state, bribes may be paid for access. Interviews with business people in Eastern Europe and Russia indicate that payoffs are frequently needed to obtain credit... In Lebanon a similar survey revealed that loans were not available without the payment of bribes.”

5Sarap (1991) in his field study in some parts of Eastern India notes that local rich and politically powerful
farmers, who may be subjected to harassment by a corrupt official (Dreze, 1990; Balmohandas et al, 1991; Jodhka, 1995). In the former case, the well-connected farmer is powerful enough to capture the subsidy meant for others. In the latter, legitimate borrowers fall prey to corrupt officials. Defaults in this case may be a result of the official’s extractiveness. Our objective is to study the second scenario.

While the above studies are useful, very little can be ascertained from them about the link between corruption and default. Given this lack of a pointed empirical study, a theoretical investigation can provide some insight and also some direction of empirical research. Keeping this objective in mind, we raise two questions: Why and when would a corrupt official like to force a borrower to default, and if so, what would be the optimal credit scheme from the government’s point of view? The first question is an issue of a positive analysis, as it may help us speculate on the repayment performance of certain types of credit schemes in a corrupt environment. The second question is important from the policy point of view. A number of studies have emphasized on the adverse consequences of corruption on the provision of public goods to the people (ranging from large farmers, teachers, and lawyers to traders) are treated on a preferential basis. This group receives bulk of the subsidized credit. However, he found that delay was much more of a problem than bribe, and delay was inversely related to the borrower’s wealth or social status (such as land-holding). He also notes that 65 percent of the borrowers in his sample defaulted on their loans. At the national level, Reserve Bank of India, reports that at 2000-01, compared to the small farmers, rich farmers received 1.90 times more loans (in amounts) from the commercial banks, but the loans outstanding from them was 2.35 times greater than that from the small farmers (Tables 52, 53, Reserve Bank of India, 2002-03). The Indian government also admitted that in 1996 it failed to recover 40 percent of agricultural loans given by state-owned banks (Government of India, 1998). Though these are aggregate figures, and loans can be of various types, we do get a feel that the problem of default really endemic.

In the context of the rural economy of India, it has been observed that corruption varies in terms of forms and intensities depending on the type of agencies that are in charge of delivering credit. While rural branches of large commercial banks follow more standardized procedures that restrict corruption, localized loan agencies can be a playground for bureaucrats, politicians and the local elite. Dreze (1990) observes that poor farmers in a North Indian village had to pay some bribes to get a loan in connection with an anti-poverty program. Another North Indian field study (Jodhka, 1995) shows how an all-powerful bureaucrat in charge of a cooperative bank can demand bribes at every step. However, this picture varies between regions. In Southern India, Balmohandas et al (1991) observes that bribery, though quite common, is not extractive to lead to default. In their survey, 45 percent of the borrowers admitted paying ‘speed money’ to the tune of about 10 percent of the loan amount, but the repayment rate was 80 percent. However, inordinate delays (averaging three months) were a serious problem faced by 40 percent of the borrowers.
services (Shliefer and Vishny, 1993), on investment in human capital (Ehrlich and Lui, 1999), and on the provision of health care and education services (Gupta et al, 2001). In a similar vein, we ask: Is it optimal for the government to reduce the provision of credit?

We consider a two-period model of credit provision under asymmetric information with the possibility that the loan official can be corrupt. The credit scheme we consider provides incentive to repay through promise of bigger loans in future, instead of demanding substantial collaterals. If the official is corrupt, he will extract bribe and cause delay to reduce the borrower’s profit, but may not necessarily wish to see the borrower default. Being less extractive now, he can sustain a stream of bribes over time, and if the government appropriately sets the loan amounts, repayment can be ensured even when the official is corrupt. Thus, we first establish that corruption, though reducing the borrower’s income need not increase the default rate.

However, this argument turns out to be valid only under full information. If the borrower has private information on his productivity, a corrupt official will have to incur ‘revelation costs’, when he tries to extort bribes. Should he wish to deal with the borrower over time (i.e. by inducing repayment), he must concede dynamic information rent to one type of the borrower. On the other hand, a short-term dealing (i.e. by inducing default) would allow him to save on large revelation cost, and therefore, in some situations default may be preferred. By modeling the interaction between the borrower and the official as a principal agent problem (or as a monopoly price discrimination problem), we are able to associate repayment with a ‘long term contract’ and default with a ‘short term contract’. We show that the probability of the official offering a short term contract instead of a long term contract to certain types of borrower - an inefficient outcome - is always positive, and cannot be driven to zero, unless corruption is altogether eliminated.

We then ask: what would be the optimal credit scheme for the government? If it wishes to restrict extraction and ensure repayment, it must reduce the size of the first period loan to curb the corrupt official’s short-term payoff. A big wedge between the credit amounts of the two periods seems necessary to align the long-term interest of the corrupt official with that of the borrower. Thus, it may be optimal to under-provide credit in the current period, if repayment is to be ensured. In some situations, however, the government can do better by permitting default by certain type of borrowers, and can ease the under-provisioning problem.
Our approach broadly follows the well-established literature on misgovernance (Shleifer and Vishny, 1993; Banerjee, 1997; Choi and Thum, 2003). While Shleifer and Vishny (1993) and Banerjee (1997) offered static analysis of bureaucratic corruption, Choi and Thum (2003) developed a dynamic version of Shleifer and Vishny. The possibility that a corrupt official may repeat extortion in future distorts entry decisions of the entrepreneurs in the current period. The official’s inability to commit to a future strategy reduces his monopoly power and his bribes from selling permits. The ratchet effect causes inefficient entry decisions. In our dynamic model also the informational ratchet effect reduces the long-term payoff of the corrupt official and induces him to prefer default, - a short-term transaction. In addition, we also observe that the default strategy is sometimes characterized by countervailing incentives, which typically reduce the revelation costs, and make default a more attractive option. We note that as long as one of these two factors is present, a model of our kind will make the official’s behavior inefficient in the sense that he would induce default.

There are several other papers that have considered bribery, red tape or harassment. Lui (1985) is one of the early contributions on queuing and bribery. Although we do not have queues, red tape has some similarity with queuing. Chaudhuri and Gupta (1996) considered a similar set up like ours in their analysis of bribery in the provision of formal credit, but their main interest is to study the interaction between formal and informal credit markets. Default is not an issue there. Saha (2001) has studied corruption in the provision of subsidy using bribe and red tape as a screening device. This approach is similar to Banerjee (1997), and is applied in this paper as well. In the present paper they are also indicators of harassment. However, harassment or extortion can be modeled in different ways. See Hindriks et al (1999), Marjit et al (2000) and Saha (2003) for different approaches to harassment in tax evasion. But these papers share features of law enforcement models (Mishra, 2002), and do not pertain to bureaucratic corruption, which is the main concern of this article.

The rest of the paper is organized as follows. In the next section we begin with the benchmark case of full information where the official also observes the borrower’s productivity. Then we move on to the asymmetric information case in Section 3, to consider the corrupt official’s behavior when all types of borrowers are productive and then derive one of our main results concerning default. In Section 4, we repeat the same exercise assuming that one type of the borrower can be unproductive. Section 5 presents a discussion of what the government’s
optimal credit program should be. The concluding section discusses limitations of our work and comments on some empirical issues.

2 The model preliminaries

2.1 The setup

Our model has two periods and two players: one loan disbursing official (principal) and a representative borrower (agent). There is also a third player, the higher authority (or simply the government), who acts like a super-principal. In the first part of our analysis, the role of the super-principal will be taken as exogenous; later we endogenize his decisions as well. The borrower belongs to a population, which by a random draw of Nature contains $p$ proportion of high productivity ($k_h$) individuals, and $(1-p)$ proportion of low productivity ($k_l$) individuals.\footnote{Nothing is lost if the population size is set to be 1.}

A high productivity individual can always convert one dollar into a sum of more than one dollar, but a low productivity individual may or may not be able to do so. This is captured through the assumption that $k_h$ can take only one value, $k_H$, which is greater than 1. On the other hand, $k_l$ can take two values: $k_L$ and $k_U$, $k_U < 1 < k_L < k_H$. When $k_U$ is realized, the individual becomes unproductive, and will invariably default on any loan he takes. Given that a borrower’s productivity is not $k_H$ (or simply $H$ type), he will be $L$ type with probability $\beta$ and $U$ type with probability $(1-\beta)$. All low productivity borrowers are assumed to have the same realization of $k_L$.

The information structure: The government would like to advance the loan only to the $H$ or $L$ types. But, neither it nor the loan official can distinguish borrower types, as the realizations of $k_h$ and $k_l$ are only privately observed by the borrowers alone. Therefore, unless the $U$ type borrowers are discouraged through a high collateral or ex post penalty, it is impossible to separate them from productive borrowers.

We reduce the informational uncertainty on the part of the official by assuming that he learns whether the realization of $k_l$ has been $k_L$ or $k_U$, but still he cannot distinguish $H$ from $L$ or $H$ from $U$. The official may have expertise (such as to conduct a market survey, or process a public signal) to learn whether he is going to deal with a $(k_H, k_L)$ distribution, or a $(k_H, k_U)$ distribution, but cannot acquire finer information about individual borrowers.
The terms of the loan: The borrower, whose loan has already been approved by the higher authority by some mechanism, is to receive a loan of \( c_1 \) dollars in the first period, and \( c_2 \) dollars in the second period, subject to the repayment of the first loan.\(^8\) The loan is interest free, but carries a penalty on default. We permit quite a general structure of penalties: \( D_1 \) on the default of \( c_1 \) and \( D_2 \) on \( c_2 \). Although we are going to emphasize on an increasing profile of penalties, \( D_1 \leq D_2 \), no \textit{a priori} restrictions are needed other than \( 0 \leq D_1 \leq c_1 \) and \( 0 \leq D_2 \leq c_2 \). These penalties can be interpreted in many ways. The simplest case is that of collaterals. Another possibility is \textit{ex post} fines, such as confiscation of household assets, or temporary withdrawal of food subsidy or health benefits etc. In the extreme case, where the borrower has no wealth whatsoever, \( D_1 \) and \( D_2 \) both can be zero.\(^9\)

Anti-corruption measures: The efficiency of the loan delivery system depends on the honesty of the loan-disbursing official. The government knows that the official will be honest with probability \( q \), and corruptible with probability \( 1 - q \). The official’s type is his private information. An honest official never takes bribe and never delays delivering the loan. But a corrupt official acts like a price-discriminating monopolist, who demands bribes and imposes red tape for each type. The red tape here simply taken as unrecorded delay, from which the official is assumed to derive utility. This means that though the loan is delivered at a later date, the official record will carry no evidence of it.\(^{10}\) We must also add that such extortions

\(^8\)We do not go into the questions of how to screen credit-worthiness of the borrowers. The government can ask the official to report their learning about \( k_1 \), and may use that information while approving the loan. For example, if the report is that \( k_1 = k_U \), a loan application may be approved only with probability \( p \). Very often eligibility for subsidized loans is centrally determined based on more observable criteria. But these issues are not important for our formulation.

\(^9\)Many micro finance organizations in their dealings with poor borrowers, retain a part of their loans as a proxy collateral, which is released only after some installments of the loan are repaid. Effectively, the loan structure becomes dynamic. We should also note that our main concern for default is only about the first period. The last period default problem can be eliminated by setting \( D_2 \) as high as \( c_2 \), and setting \( D_2 \) high may be feasible. After all, the credit subsidies offered in earlier periods could create wealth against which future loans may be issued.

\(^{10}\)That the official derives positive utility from red tape is not essential. Red tape can be a pure waste as in Banerjee (1997), for instance. However, in developing countries where government employees are often poorly paid, their disutility from labor takes a toll on the speed of work. The entire Indian literature cited before found that delay and bureaucratic procedures are serious problems. Though in some cases harassment takes the form of inordinate delay, it is not clear whether large bribes can significantly reduce the delay factor.
are possible only if the official can credibly deny the loan if bribes are not paid. Therefore, we must assume that the official is endowed with some power to stop the loan, and he may abuse this power.

The welfare-minded government observes neither bribe nor red tape, but only the records of loan deliveries and subsequently the records of repayment or default. But being aware of the possibility of corruption, it randomly investigates the official’s activities. The probability of investigation in the first period is always \( \mu (\mu < 1) \). In the second period, it is again \( \mu \), if investigation was not conducted earlier, or if investigation did not show any evidence of corruption. When corruption is detected, the official is fined by \( F \) dollars, and then in the second period, he will be investigated again, this time with probability 1, leading to a severe penalty \( F_2 \), if found to be still extorting bribes. The fine \( F_2 \) is high enough to deter him from doing so.\(^{11}\)

We assume that investigation always uncovers bribery, but cannot establish a link between bribery and default, if a default is subsequently observed. In other words, the investigating agency learns only about side-payments, but cannot determine what type of borrower has made these payments. Therefore, if a default is observed, and if \( U \) type of agents were present in the pool of borrowers, then the official cannot be penalized beyond a fine of \( F \) dollars. But if the \( U \) type borrowers were excluded by requiring a high collateral \( (D_1) \), then any default would automatically imply that the official has been over-extractive. In that case, we assume that the official will be fined next period very severely by \( F_2 \) dollars.

Thus, to the government the degree of corruption matters. It is particularly severe on repeated corruption and over-extractive corruption (causing default) when they are evident. In contrast to such severe punishments, \( F \) is assumed to be mild. A punishment for first time bribery may just involve a cut in salary, an adverse comment on his service record, delay in promotion etc.\(^{12}\) We assume for simplicity that investigation takes place before the borrower

\(^{11}\)This is similar to transferring the official to a different location or a different task that offers no bribe opportunities. Transfer of officials is actually a common practice in the Indian bureaucracy. While such transfers reduce the official’s incentive to be corrupt, there is no guarantee that the next official will not be corrupt. Therefore, transfers do not protect the customers or agents in this set up, unless the threat of transfer really bites.

\(^{12}\)Two assumptions are implicit: First, the borrowers cannot increase the probability of investigation by reporting corruption. The higher authority may not find such reporting backed by hard evidence. Second, only one official is given the sole charge of disbursing loans to the whole group. Allowing multiple servers may
decides to default or repay.

The utility function of the corrupt official is:

\[ u = B + 2\sqrt{t} - \mu F, \]

where \( B \) refers to bribe and \( t \) to red tape. An honest official derives no separate utility other than from his salary. We normalize utility from salary to be zero.

The borrower has a CRS technology, and there is no uncertainty in production. In the presence of red tape and bribe, the borrower’s gross (or pre-repayment) profit in period \( i \) is

\[ R_i = k(c_i - B) - t \]

where \( k \in \{k_H, k_L, k_U\} \) is the borrower’s private information. The reservation payoffs of the borrower and the official are both zero.

Now we turn to the issue of default. It is clear that smaller the size of \( D_1 \) greater is the chance of willful default. This can be countered by making the second period loan \( (c_2) \) attractive. At the end of the second period, there is always a problem of default unless \( D_2 \) is as high as \( c_2 \). When \( D_2 = 0 \), the second period loan becomes a pure transfer.

When \( D_1 < c_1 \), the borrower decides to repay the first loan, only if the following inequality holds:

\[ k(R_1 - c_1) + (R_2 - D_2) \geq k(R_1 - D_1). \]

The left hand side of this inequality represents the total two period net profits of the borrower (for types \( H \) or \( L \)) when he repays. The first period net profit \( (R_1 - c_1) \) is reinvested in period 2 and it becomes \( k(R_1 - c_1) \). The second period net profit is \( (R_2 - D_2) \). The right hand side represents the total profits when he defaults in period 1. This inequality reduces to (for any given type \( j \))

\[ R_2^j \geq D_2 + k_j(c_1 - D_1), \quad j = H, L. \]

where \( R_2^j \) should be seen as a promise by the official to compensate the \( j \)-type borrower in future for his current loss from ‘not defaulting’, \( k_j(c_1 - D_1) \), plus the second period default cost \( D_2 \). We assume that such promises will be kept. Here, the official is assumed to have elicit more information, but then collusion among the servers is to be ruled out. We abstract from such issues.
informal means of commitment. He may also worry about his reputation, as he deals with a number of borrowers.\textsuperscript{13} Thus, inequality (1) specifies the necessary incentive for repayment, and we may refer to it as the \textit{repayment-incentive} condition. To this we must add a \textit{feasibility} condition:

$$R_j^i \geq c_1, \quad j = H, L.$$  \hfill (2)

These two conditions must be fulfilled to induce repayment. Since the repayment decision is taken in the subgame, the borrower knows whether the official is corrupt or honest, and whether the corruption has been detected or not. Depending on the history, the expression of $R^2$ will vary.

If the official is honest (or if corruption has been uncovered), $R^2$ will be given by $kc_2$ (ignoring subscript $j$), and condition (1) becomes:

$$(c_2 - c_1) \geq \frac{D_2}{k} - D_1, \quad k \in \{k_L, k_H\}.$$  \hfill (3)

In this case, The borrower’s payoff at the end of the second period becomes:

$$\bar{\Pi}(k) = k[(k-1)c_1] + (kc_2 - D_2), \quad k \in \{k_L, k_H\}.$$  \hfill (4)

The first bracketed term is the first period profit, which after reinvestment in the second period becomes $k[(k-1)c_1]$. The second term is the second period profit.

For a $U$ type borrower the repayment constraint is irrelevant, but what matters most is whether $D_1 < k_U c_1$ or not. If $D_1 \geq k_U c_1$, he does not take the loan. But if $D_1 < k_U c_1$, he clearly benefits from the loan as his profit becomes:

$$\bar{\Pi}(k_U) = k_U(k_U c_1 - D_1).$$  \hfill (5)

In the case of a corrupt official (who is not yet caught), $R^2$ becomes: $R^2 = k(c_2 - B_2) - t_2$ and condition (1) changes to,

$$(c_2 - c_1) \geq \left[\frac{D_2}{k} - D_1\right] + B_2 + \frac{t_2}{k},$$  \hfill (6)

where $B_2$ and $t_2$ are to be optimally chosen by the official. Note that the required gap between $c_2$ and $c_1$ has increased.

\textsuperscript{13}It is not rare to see in rural India, where the people often take matters in their own hands, a corrupt official is punished by the locals for his over-extractive behaviors, or for dishonoring promises. Thus, local norms can also play a role.
But condition (6) is not enough to ensure repayment. An additional condition needs to be satisfied to ensure that the corrupt official also prefers repayment to default. This condition is far from obvious. We need to compare the official’s expected payoff from the two choices and then derive the condition for repayment. Our main task is to determine this choice of the official when information is asymmetric.

The structure of information will be clear from the following description of the game that we are going to consider:

*Stage 1:* The government decides on the loan amounts and penalties on default.

*Stage 2:* The Mother Nature chooses the productivity of the borrowers, which they observe privately. The official learns whether $k_l$ is realized as $k_L$ or $k_U$, but cannot distinguish between a high and a low type borrower.

*Stage 3:* A borrower comes to the official to receive the loan sequence ($c_1, c_2$). If the official is honest, the loan is given right away. If the official is corrupt, the borrower self-selects from a menu of bribe demand and red tape. At this point, the official may be investigated. If investigated and found corrupt, he has to pay a fine $F$. Subsequently, if the borrower defaults, he loses $D_1$ dollars, and the game ends. If he repays, the game goes to the next period.

*Stage 4:* In the next period, if the corrupt official is under vigilance, the borrower receives the loan instantly without paying any bribe. If the official is not under vigilance, the borrower is again subjected to bribery and red tape. An honest official, as always, disburses the loan efficiently. The game ends with default, if $D_2 < c_2$. Otherwise, the borrower repays.

### 2.2 Symmetric information

We begin with the benchmark case of symmetric information, where the borrower’s productivity is observed by both the official and the borrower (but not by the government). First the case of an honest official. Assuming that condition (1) holds, the borrower is immediately served, only if he is either $k_H$ or $k_L$. A $U$ type is not served by an honest official.

With a corrupt official the story changes. He may serve a $U$ type to extract bribe. But dealings with $H$ or $L$ are more attractive as the potential surplus is larger. When he meets one of these two types, he makes an offer \{$(B_1, t_1), (B_2, t_2)$\} that maximizes his total payoff (without discounting): $U = u_1 + u_2 = [B_1 + 2\sqrt{t_1} - \mu F] + (1 - \mu) [B_2 + 2\sqrt{t_2} - \mu F]$, subject to the repayment incentive condition (6) and the repayment feasibility condition (2).
It is straightforward to derive:

\[ B_2^* = (c_2 - c_1 + D_1) - (k + \frac{D_2}{k}), \quad t_2^* = k^2. \]

\[ B_1^* = c_1 \frac{(k - 1)}{k} - k, \quad t_1^* = k^2 \]

Note that the official’s payoff in the first period is

\[ u_1^*(k) = c_1 \left(\frac{k - 1}{k}\right) + k - \mu F, \]

and in the second period is

\[ u_2^*(k) = (c_2 - c_1 + D_1) + k - \frac{D_2}{k} - \mu F. \]  \hspace{1cm} (7)

We assume that \( u_2^*(k) > 0 \) for both \( k_H \) and \( k_L \). His total utility over two periods is:

\[ U_R(k) = u_1^*(k) + (1 - \mu)u_2^*(k) \]

\[ = \frac{(k - 1)}{k} c_1 + k + (1 - \mu) \left[(c_2 - c_1 + D_1) + k - \frac{D_2}{k}\right] - \mu(2 - \mu)F. \]

The official’s offers can be seen as a long-term contract that he is able to commit to by some informal means. Alternatively, the second period offers are to be seen as a promise that he will honor in future. If he deals with a large number of borrowers, he should worry about reputation and thus, will not go back on his promise. This long term contract is also replicable through a sequence of short term contracts, as long as the official is required to honor his promise.

From the above offers, the borrower receives over two periods:

\[ \Pi(k) = (1 - \mu)k(c_1 - D_1) + \mu(kc_2 - D_2). \]  \hspace{1cm} (8)

Having met a corrupt official, the borrower will get only zero net profit in the first period, and in the second period, he will get his promised payoff \( k(c_1 - D_1) \) (follows from (1), or the first best payoff \( k(c_2 - D_2) \) which occurs following an investigation.

This is the borrower’s payoff when the official induces repayment. If he were to make the borrower default, or had he met a \( U \) type (when \( D_1 < k_U c_1 \)), his problem would reduce to maximizing \( u_1 = B_1 + 2\sqrt{t} - \mu F \) subject to \( k(c_1 - B_1) - t_1 \geq D_1 \). This yields to a short term contract as

\[ B_1^* = c_1 - k - \frac{D_1}{k}, \quad t_1^* = k^2 \]

leading to zero profit for the borrower and the following to the official:

\[ U_D(k) = c_1 + k - \frac{D_1}{k} - \mu F. \]
So now for the official to prefer repayment, $U_R$ must exceed $U_D$, which requires:

$$(c_2 - c_1) \geq \left\{ \frac{D_2}{k} - D_1 \right\} + \left\{ \frac{(c_1 - D_1)}{k(1 - \mu)} - k + \mu F \right\}.$$  \hspace{1cm} (9)

This is the new repayment incentive condition, which is stronger than no-willful-default conditions (3) and (6). This condition says that $(1 - \mu)u_2^*(k) \geq \frac{c_2 - D_1}{k}$. That is the official’s second period expected payoff must be sufficiently large. If both $H$ and $L$ are to be induced to repay (under full information), this condition must be set for type $L$, which is assumed below.

**Assumption 1**: $(c_2) \geq c_1 + \left\{ \frac{D_2}{k_L} - D_1 \right\} + \left\{ \frac{(c_1 - D_1)}{k_L(1 - \mu)} - k_L + \mu F \right\}.$

In addition, bribes are assumed to be postive (for each type), and for simplification, we also assume:

**Assumption 2**: $1 < k_L < k_H < 2.$

**Observation 1**: Suppose the official could observe the borrower’s productivity, and Assumption 1 holds, and $D_2 = c_2$. Then regardless of whether the official is honest or corrupt, the loan is fully repaid in both periods by $H$ and $L$ type borrowers.

**Welfare analysis**: How do various loan parameters affect the players’ welfares? The total (two-period) expected profit of a given type of borrower (before meeting the official) is:

$$E\Pi(k) = q\bar{\Pi}(k) + (1 - q)\Pi(k)$$
$$= q[k(k - 1)c_1 + kc_2 - D_2] + (1 - q)[(1 - \mu)k(c_1 - D_1) + \mu(kc_2 - D_2)].$$

As we have already noted, a corrupt official (with monopoly power) can fully extract the second period surplus from the borrower, simply by giving him what he could have got by defaulting in the first period. The only hope of getting something from the second period lies with the chance of corruption detection.\(^{14}\)

Therefore, the second period loan parameters $c_2$ or $D_2$ will have very little effect on the borrower’s welfare, unless $q$ is sufficiently high, and/or $\mu$ is high.

**Observation 2**: Both $c_1$ and $c_2$ positively affect the expected profit of the borrower. But $c_2$ has relatively greater impact, if $\mu \geq 1/2$, or $\mu < 1/2$ and $q > \frac{(1 - 2\mu)}{(1 - 2\mu + 2 - k^2)}$. On the other\(^{14}\)

\(^{14}\)Thus, the anti-corruption measures here are working in a way similar to giving some bargaining power to the borrower.
hand, the effects of $D_1$ and $D_2$ are adverse. In absolute terms, the effect of $D_2$ is stronger if $\mu \geq \frac{k}{k+1}$, or if $\mu < \frac{k}{k+1}$ and $q > \frac{k-\mu(k+1)}{1+k-\mu(k+1)}$.

These results are obvious, once we take the derivatives of $E\Pi$ with respect to the relevant variables. Interestingly, note that the effect of $D_1$ is felt only in the event of undetected corruption. But then $D_2$ becomes irrelevant. Thus, the two ‘collateral’ parameters (reflecting the degree of loan securitization) work in mutually exclusive situations. Now we look at the corrupt official’s payoff.

Observation 3: Assuming that the corrupt official induces repayment, his expected utility will increase if $c_2$ or $D_1$ increases. But an increase in $D_2$ will adversely affect him (because it reduces the second period bribe). On the other hand, the effect of an increase in $c_1$ is ambiguous. But more interestingly, an increase in the probability of investigation hurts him more when he induces repayment, than when he induces default.

The Effects of $c_2$ and $D_1$ are obvious. An increase in $c_1$ increases bribe in the first period, but reduces the second period bribe. The positive effect dominates, only if the prospect of a second period bribe is lower, which is possible only if $\mu$ is large enough. So only with a sufficiently higher probability of detection, the official would prefer to see a larger size of the loan in the first period. But the effect of an increase in $\mu$ is interesting. With greater $\mu$ the expected penalty from the first period increases, and the second period expected payoff also falls. This makes the repayment strategy quite unattractive. On the other hand, under the default strategy, the effect of a rise in $\mu$ is felt only in the first period. Formally, $\frac{\partial U_D}{\partial \mu} = -F$, and $\frac{\partial U_R}{\partial \mu} = -F - \{(c_2 - c_1 + D_1 \frac{D_2}{k} + k - \mu F\} + (1 - \mu)F\}$. The first term inside the bracket is positive by assumption. Hence the adverse effect of $\mu$ on $U_R$ is stronger.

We note that stricter anti-corruption measures may not necessarily make the prospect for an efficient outcome higher. In our model, the anti-corruption measures do not affect the official’s marginal calculations. Instead they affect only the total payoff. In this case, the total payoff is much more adversely affected when the official induces repayment. This observation will carry over to the case of asymmetric information as well, and will have a bearing on the government’s design of the optimal credit program.

How does the government decide on its optimal credits? While we postpone a formal discussion till Section 5, it is clear that in a symmetric information environment as long as
the credit amounts satisfy Assumption 1, repayment is ensured for both $H$ and $L$. The levels of credit are then to be decided on the basis of social welfare considerations. Since Assumption 1 is crucial for repayment, by comparing it with (9) we can say that the divergence between the two credit amounts will be greater compared to a corruption-free environment. Default by a $U$ type borrower can also be prevented, if $D_1$ can be raised to $k_{UC_1}$. But this may not always be possible, especially when the borrower has little wealth. An observation of default in this case will imply that the official has knowingly served an unproductive borrower. If the anti-corruption measures are strong enough to deter him from doing so, default again is prevented.

3 Asymmetric information

3.1 The case of $k_L$

In the absence of complete information, nothing changes for the honest official, except that he cannot turn away a $U$ type. But first we shall consider the case where $k_L$ is realized instead of $k_U$. The honest official immediately disburses the loan, and both types of agents receive their first best payoff. Repayment occurs with certainty at the end of the first period.

But a corrupt official, now constrained by asymmetric information, would like to screen the borrowers by offering a menu of bribes and red tapes to induce self-selection with the specific objectives of inducing default or repayment.

He has four possible strategies - ‘both types repay’, ‘both types default’ and ‘only one type defaults’; of these only two strategies - ‘both types repay’ and ‘only $L$ defaults’- become relevant. It turns out that forcing $H$ alone to default while $L$ repays is not feasible, and the strategy ‘both types default’ is dominated by ‘both types repay’ under some reasonable assumptions.

3.1.1 Both types repay: Strategy $R$

Under the strategy of ‘both types repay’, the official may face the informational uncertainty in both periods. Therefore, he would like to screen the borrower through separating offers in
the first period, and then in the second period have the full information payoff.\textsuperscript{15} He will offer a menu of long term contracts, $\{(B_{1}^{H}, t_{1}^{H}), (B_{2}^{H}, t_{2}^{H})\}$ and $\{(B_{1}^{L}, t_{1}^{L}), (B_{2}^{L}, t_{2}^{L})\}$, from which the borrower will self-select depending on whether he is $H$ or $L$ respectively. Since both types are uniformly treated, this is a case of a uniform (long-term) contract.

These offers should maximize the official’s two-period expected utility subject to a set of constraints for each type. The set of constraint consists of the repayment incentive condition (6), repayment feasibility condition (2), incentive compatibility condition and individual rationality condition. Given that we are going to consider only separating offers, the second period problem is identical to the full information case. $(B_{1}^{H}, t_{1}^{H})$ and $(B_{2}^{L}, t_{2}^{L})$ will be same as before giving rise to $u_{2}^{*}(k_{H})$ and $u_{2}^{*}(k_{L})$ as given by equation (7). In expected terms, his second period payoff is $Eu_{2}^{*} = pu_{2}^{*}(k_{H}) + (1-p)u_{2}^{*}(k_{L})$. Once again, the second period offers are to be seen as a promise that will be honored.

Therefore, we need to focus only on the first period problem which will concern mainly incentive compatibility ($IC$), and repayment feasibility ($RC$) constraints. It turns out that individual rationality ($IR$) constraints are automatically satisfied if $RC$-s are met. The repayment incentive constraint (6) is satisfied by the second period offers.

We begin with the incentive compatibility conditions. Define $(R_{i,j}^{H}, R_{i,j}^{L})$ as the sequence of gross profits a type $j$ borrower gets by truthfully revealing himself. Similarly, $(R_{i,j}^{H}, R_{i,j}^{L})$ is the sequence of gross profits a type $i$ borrower gets by misrepresenting as type $j$. So we can write the incentive compatibility constraints for type $H$ and $L$ as:

$$(R_{1}^{H} - c_{1})k_{H} + (R_{2}^{H} - D_{2}) \geq (R_{1}^{H} - c_{1})k_{H} + (R_{2}^{H} - D_{2})$$

$$(R_{1}^{L} - c_{1})k_{L} + (R_{2}^{L} - D_{2}) \geq (R_{1}^{L} - c_{1})k_{L} + (R_{2}^{L} - D_{2})$$

Note that $R_{2}^{H}$ and $R_{2}^{L}$ are to be given by condition (6) with optimal bribes and red tapes. But what about $R_{1}^{H}$ and $R_{1}^{L}$? First, consider $R_{1}^{H}$ that $H$ can earn in period 2 by misrepresenting in period 1:

$$R_{2}^{H} = k_{H}(c_{2} - B_{2}^{L}) - t_{2}^{L} = \frac{k_{H}}{k_{L}}D_{2} + (k_{H} - k_{L})k_{L} + k_{H}(c_{1} - D_{1}).$$

This is strictly greater than his gross profit under truthfulness: $R_{1}^{H} = D_{2} + k_{H}(c_{1} - D_{1})$. This means that $H$ has dynamic incentive to misrepresent his type.

\textsuperscript{15}To avoid potential complications of pooling vs. separating offers, we simply assume that separating offers exist, and they are optimal. In fact, it will be clear later on, that our key result on the official’s incentive to inflict default will be stronger, if offers are pooling.
Substituting the expressions for $R_{2}^{H,H}$ and $R_{2}^{H,L}$ we rewrite the incentive constraint of $H$ as:

$$R_{1}^{H,H} \geq R_{1}^{H,L} + \left[\frac{(k_{H} - k_{L})}{k_{H}k_{L}}(D_{2} + k_{L}^{2}\rho)\right]$$  \tag{10}

The presence of the second term indicates dynamic incentives. In other words, to reveal his type $H$ must be given a sufficiently large payoff in the first period to cover his long term gains from untruthful behavior.

Now consider $L$’s second period payoff after he misrepresents in the first period. This is $R_{2}^{L,H} = \frac{k_{L}}{k_{H}}D_{2} + (k_{L} - k_{H})k_{H} + k_{L}(c_{1} - D_{1})$ which is strictly less than his payoff under truthful revelation: $R_{2}^{L,L} = D_{2} + k_{L}(c_{1} - D_{1})$. This implies that having misrepresented, $L$ will default and opt out of the second stage game. Therefore, his incentive compatibility condition reduces to:

$$R_{1}^{L,L} \geq R_{1}^{L,H}.$$  \tag{11}

Now we state the official’s problem (call it problem $P$) where $(B, t)$ without a time subscript will refer to the first period choice:

$$\text{Max } V = p(B^{H} + 2\sqrt{t^{H}}) + (1 - p)(B^{L} + 2\sqrt{t^{L}}) + (1 - \mu)Eu_{2}^{*} - \mu F$$

subject to

$$(IC_{H}) : \quad k_{H}B^{H} + t^{H} \leq k_{H}B^{L} + t^{L} - \frac{(k_{H} - k_{L})(D_{2} + k_{L}^{2}\rho)}{k_{L}k_{H}}$$

$$(IC_{L}) : \quad k_{L}B^{L} + t^{L} \leq k_{L}B^{H} + t^{H}$$

$$(RC_{H}) : \quad k_{H}B^{H} + t^{H} + c_{1} \leq k_{H}c_{1}$$

$$(RC_{L}) : \quad k_{L}B^{L} + t^{L} + c_{1} \leq k_{L}c_{1}$$

**Proposition 1**  
(a) Assuming that separating offers exist, the official makes the following offers:

$$B^{H} = \left[c_{1}\frac{(k_{H} - 1)}{k_{H}} - k_{H}\right] - \left[\theta(c_{1} + \frac{k_{L}^{2}}{(1 + \rho k_{L})^{2}})\right] - \left[\frac{\theta(D_{2} + k_{L}^{2}\rho)}{k_{H}}\right]$$  \tag{11}

To see this, write $L$’s incentive compatibility condition as: $(R_{1}^{L,L} - c_{1})k_{L} + (R_{2}^{L,L} - D_{2}) \geq (R_{1}^{L,H} - D_{1})k_{L}$, and then substitute the expression for $R_{2}^{L,L}$.  \footnote{To see this, write $L$’s incentive compatibility condition as: $(R_{1}^{L,L} - c_{1})k_{L} + (R_{2}^{L,L} - D_{2}) \geq (R_{1}^{L,H} - D_{1})k_{L}$, and then substitute the expression for $R_{2}^{L,L}$.}
\[ t^H = k_H^2 \]  
\[ B^L = c_1 \frac{(k_L - 1)}{k_L} - \frac{k_L}{(1 + \rho \theta k_L)^2} \]  
\[ t^L = \frac{k_L^2}{(1 + \rho \theta k_L)^2} \]  

where

\[ \theta = \frac{1}{k_L} - \frac{1}{k_H}, \quad \rho = \frac{p}{1 - p}. \]

(b) The first period net (post-repayment) profits of the borrower are: \( \pi_1^H = [(c_1 - B^L)(k_H - k_L) + \theta(D_2 + k_L^2)] > 0 \) and \( \pi_1^L = 0 \). Over two periods, the expected net profits are:

\[ \Pi^H_R = k_H \pi_1^H + (1 - \mu)(c_1 - D_1)k_H + \mu[k_Hc_2 - D_2] \]
\[ \Pi^L_R = (1 - \mu)(c_1 - D_1)k_L + \mu[k_Lc_2 - D_2]. \]

For proof see Appendix A.

The proposition shows that compared to the full information case, the \( H \) type will earn greater profits by paying less bribe, whereas the \( L \) type will pay a higher bribe, and get compensated through a smaller red tape with no improvement in profits. This follows from the fact that of the four constraints only \( RC_L \) and \( IC_H \) will bind. By comparing the two repayment constraints it can be seen that (since \( k_H > k_L \) ) any offer that makes \( L \)'s net profit zero, will make \( H \)'s net profit strictly positive. Thus, quite predictably \( H \) will earn information rent.

Equation (11) has three (bracketed) terms. The first term is the full information bribe. The second term is the static and the third term is the dynamic component of information rents. We must note that the second term has a lower bound \( \theta c_1 \) and the third term is completely invariant to \( p \). This implies that strategy \( R \) is not only plagued with high rents, but the rents also persist even if \( p \to 1 \). Consequently, there will be a discontinuity at \( p = 1 \).

We should also note that here we observe greater delay for the high productivity borrower. This may appear contradictory to the results of queuing models, such as Lui (1985). In a queuing model, the cost of waiting determines the agent’s type, and a high cost type will pay higher bribe to be served earlier than a low cost type. The same result can be obtained also from a screening model, if the agent’s type is given by cost of waiting (See Saha (2001) for example). But in the present context, the borrowers’ types differ in terms of their valuations.
of credit, and not in terms of the cost of delay. A $H$ type values credit more than a $L$ type, which implies that the relative cost of delay is lower to $H$, than to $L$. Hence the high type faces greater delay.

Space for Figure 1.

Figure 1 illustrates the optimal offers. The full information offers are denoted as $H_F$ and $L_F$, which are given by tangency between the borrower’s linear iso-profit curves and the corrupt official’s indifference curves. Similarly, the asymmetric information offers are denoted as $H_A$ and $L_A$. Note how the official distorts the full information offers to achieve separation. Since bribe is more expensive to $H$ than to $L$, he extracts more bribe from $L$, while moving along $RC_L$. However, to make $H$ indifferent between his offer and that of $L$, the official has to give $H$ a discount in terms of bribe reduction from his full information offer. This discount must have two parts: one for each period. If no rents were to be given for the second period, a point like $H'$ would have been optimal. Due to the dynamic rent $H_A$ must lie above or outside $RC_L$.

The official’s (undiscounted) two-period expected utility is:

$$V_R = c_1 \frac{(k_L - 1)}{k_L} + (1 - \mu) [p u_2^* (k_H) + (1 - p) u_2^* (k_L)] + \left[p k_H + (1 - p) \frac{k_L}{1 + \rho \theta k_L} \right] - p \theta (D_2 + \frac{k_2^2}{k_H}) - \mu F$$

where $u_2^* (k_H)$ and $u_2^* (k_L)$ are given by (7). It can be shown that given $F$ not too large, $V_R$ is positive, strictly convex and continuous at all $p \in [0, 1)$. It is also increasing in $p$, if $\mu \leq \frac{(k_H - 1)}{k_H}$.

3.1.2 Type L Defaults: Strategy D

Now the official wishes to make the $L$ type default, while $H$ repays. In so doing the he should worry about whether $D_1 < k_{UC_1}$ or $D_1 \geq k_{UC_1}$, because the former provides an effective cover for over-extraction with a smaller level of expected punishment. On the other hand, if $D_1 \geq k_{UC_1}$, no $U$ type is expected to take the loan, and therefore, an observation of default will invite a severe penalty ($F_2$ dollars) in the next period. However, such penalties only affect the official’s total payoff and not his marginal calculations vis-a-vis bribes and red tapes.
Regarding the bribes and red tapes, we observe an interesting possibility. To end up in default $L$ must suffer a greater cost, whereas to be able to repay $H$ must bear a smaller cost. This may encourage the $L$ type to imitate $H$. This will indeed be the case if $D_1$ is below a critical level, as specified in the following assumption:\footnote{17}{Given Assumption 3, the $IR_L$ curve will lie above the $RC_H$ curve and the incentive to misrepresent will shift from $H$ to $L$. However, if $D_1$ exceeds this critical level, the information rent may begin to dissipate. See discussions in Section 6.}

**Assumption 3:** $D_1 < (1 + k_L - k_H)c_1$.

In the official’s problem ($P$) three changes are to be made. First, the expected payoff of the second period changes for the official. Second, the constraint $RC_L$ is to be replaced by the individual rationality condition (with default). Third, no longer can $H$ exploit his private information beyond the first period. Moreover, should $H$ misrepresent, he must default (because $L$ is expected to default). Due to these considerations, $H$’s incentive compatibility condition reduces to:\footnote{18}{To see this write his incentive compatibility condition as: $(R_{1H,H}^{H,H} - c_1)k_H + R_{2H,H}^{H,H} - D_2 \geq (R_{1H,L}^{H,L} - D_1)k_H$, and then substitute the expression for $R_{2H,H}^{H,H}$ from (1).}

$$R_{1H,H}^{H,H} \geq R_{1H,L}^{H,H}.$$  

The official is going offer a mixed contract, - a long term contract to type $H$ and a short term contract to $L$. As before the second period offers for $H$ are simply given by full information offers $(B_H^2, t_H^2)$, and what remains to be solved are the first period offers for both $H$ and $L$.

Now assuming $D_1 < k_U c_1$, we state the official’s problem, which is modified as ($P'$):

Max $V = p(B_H^H + 2\sqrt{t_H^H}) + (1 - p)(B_L^L + 2\sqrt{t_L^L}) + (1 - \mu)p\mu_L^2(k_H) - \mu F$

subject to $IC_L$ and $RC_H$ as in problem $P$ and

$$(IC_L) : \quad k_H B_H^H + t_H^H \leq k_H B_L^L + t_L^L$$

$$(IR_L) : \quad k_L B_L^L + t_L^L + D_1 \leq k_L c_1$$
In this problem only $RC_H$ and $IC_L$ will bind. Consequently, $L$ will earn information rent raising its profit above $D_1$, but still he will not be able to repay.

**Proposition 2** (a) Define $p^c = \theta k_L \left[ \frac{\sqrt{c_1(k_H-1)}}{\sqrt{c_1(k_H-1)}-k_L} \right]$. The official’s optimal offers are as follows:

For $p \leq p^c$,

$$B^H = 0, \quad t^H = c_1(k_H - 1) \quad (18)$$

$$B^L = \frac{c_1(k_H - 1)}{k_L} - k_L, \quad t^L = k_L^2 \quad (19)$$

and for $p > p^c$,

$$B^H = \frac{c_1(k_H - 1)}{k_H} - k_H \left( \frac{\rho}{\rho - \theta k_H} \right)^2 \quad (20)$$

$$t^H = k_H^2 \left( \frac{\rho}{\rho - \theta k_H} \right)^2 \quad (21)$$

$$B^L = \frac{c_1(k_H - 1)}{k_H} - k_L + k_H^2 \theta \left( \frac{\rho}{\rho - \theta k_H} \right)^2 \quad (22)$$

$$t^L = k_L^2 \quad (23)$$

(b) Consequently, the first period net profits of the borrower are $\pi^H_1 = 0$, and

$$\pi^L_1 = c_1(1 + k_L - k_H) - D_1 \quad if \; p \leq p^c$$

$$= B^H(k_H - k_L) + [c_1(1 + k_L - k_H) - D_1], \quad if \; p > p^c.$$ 

and the total expected net profits are:

$$\Pi^H_D = \mu(k_Hc_2 - D_2) + (1 - \mu)(c_1 - D_1)k_H \quad (24)$$

$$\Pi^L_D = \pi^I_1 k_L > 0. \quad (25)$$

For proof see Appendix B.

The present case contrasts the earlier one in several respects. First, corner solutions are a possibility. At low values of $p$, it pays off to raise the bribe from $L$ at the expense of maximum distortion in $H$’s offer. Here, again we note a similar pattern of rent persistence. At all $p \in (0,p^c)$, $L$’s information rent ($\pi^L_1$) remains constant and strictly positive. This is, however, true as long as $D_1 < c_1(1 + k_L - k_H)$ (see Proposition 2 part (b)). Second, both types are strictly worse off as compared to the repayment case; $L$ is forced to default, and $H$ just manages to pay back.
These points are illustrated in Figure 2. Here, the IR_L line lies above RC_H (because of Assumption 3), indicating the informational advantage of L. For low enough p, the official is forced to go to the corner at c_1(k_H - 1). At higher values of p, separation occurs in the usual manner as shown by points H_A and L_A.

But the most important contrast is the reversal of incentive to misrepresent, which allows the low type to earn information rent. Reversal of incentives is a key feature in countervailing incentives models where the reservation utility of the (privately informed) agent is type-dependent. For example, in the regulation literature (see Lewis and Sappington, 1989; Maggi and Rodriguez-Clare, 1995) this reversal takes place as the agent’s type exceeds a critical value (that lies inside the continuous support of the type’s distribution function). In the context of incentive bribes Saha (2001) observes countervailing incentives due to a change in an exogenous variable. But in the present context, reversal of incentives results from the principal’s initial choice of strategy, and to our best knowledge this type of countervailing incentives has not been identified before in the context of corruption.

However, countervailing incentives arise only at low values of D_1, which has been specified in Assumption 3. At higher values of D_1 this need not be the case. In Appendix D, we present such a case. It may also appear that this is primarily a problem of separating equilibrium. If the official offers a pooling contract in the first period, say pegging R_L^1 at D_1, forcing L to default without any information rent, can this contract implement the strategy D'? The answer is ‘no’, as long as D_1 satisfies Assumption 3. In this case, the pooled offer will also force H to default. On the other hand, if D_1 > c_1 k_L / k_H, the pooled offer will allow H to repay, but the official can do better by offering a separating offer (which we show in Appendix D).

Now we write the official’s expected utility under the strategy D:

\[
V_D = p \left( 2\sqrt{c_1(k_H - 1) + (1 - \mu)u^*_2(k_H)} \right) + (1 - p) \left( \frac{c_1(k_H - 1)}{k_L} + k_L \right) - \mu F. \tag{26}
\]

The pooled offer is B = c_1 - k_L - \frac{D_1}{k_L} and t = k_L^2. This will set R_L^p = D_1 and R_H^p = \frac{k_H}{k_L} D_1 + (k_H - k_L) k_L. Now check that R_H^p > c_1 if D_1 > c_1 \frac{k_L}{k_H} - (k_H - k_L) \frac{k_L^2}{k_H}. But this cannot be true, if Assumption 3 holds. c_1 \frac{k_L}{k_H} - (k_H - k_L) \frac{k_L^2}{k_H} > c_1(1 + k_L - k_H), if c_1 > \frac{k_L^2}{k_H - 1} > \frac{k_L^2}{k_H - 1}, which is assumed to hold for nonnegativity of bribes. Therefore, R_H^p < c_1.
For $p > p^c$,

$$V_D = \frac{c_1(k_H - 1)}{k_H} + p(1 - \mu)u_2^*(k_H) + \left(\frac{p - k_H \rho}{\rho - \theta k_H} + (1 - p)k_L\right) - \mu F.$$  \hspace{1cm} (27)

Given $F$ not too large, $V_D(.)$ is strictly positive, convex, and continuous at all $p \in (0, 1]$. The segment given by (27) is strictly convex. We assume that $V_D$ is also increasing in $p$.\textsuperscript{20}

So far we assumed that $D_1 < k_{UC1}$. What will be the case if $D_1 \geq k_{UC1}$, while still maintaining Assumption 3? The official’s optimal offers do not change, but he will face a greater penalty in the following way. Regardless of whether he was fined in the first period or not, an incidence of default will attract a high penalty $F_2$. So this additional penalty makes his payoff smaller:

$$\tilde{V}_D = V_D(p) - (1 - p)F_2.$$  \hspace{1cm} (28)

Needless to say, $\tilde{V}_D(p)$ has the same derivative property as $V_D$. But since $F_2$ is large, it is possible that at some low values of $p$ (say $p \in [0, p_f]$) $\tilde{V}_D$ remains non-positive, in which case the official is better off by not playing the strategy $D$. Thus setting $D_1 \geq k_{UC1}$ can be seen as an effective way of curbing extractive corruption as long as $p$ is small. But its effectiveness disappears as $p$ is sufficiently close to 1. In this case, $\tilde{V}_D$ is close to $V_D$.

Before we proceed to the next section, it will be useful to consider also the strategy of forcing both types to default- strategy $BD$. Formulating the problem as a one period problem, we can check that type $H$ will receive information rent (but only for one period), and the menu of offers will be similar to that under the strategy $R$. The corrupt official’s expected payoff will be:

$$V_{BD}(k_L, k_H) = (c_1 - \frac{D_1}{k_L}) + pk_H + (1 - p)\left(\frac{k_L}{1 + \rho \theta k_L}\right) - \mu F$$  \hspace{1cm} (29)

if $D_1 < k_{UC1}$ and

$$\tilde{V}_{BD}(k_L, k_H) = V_{BD}(k_L, k_H) - F_2$$  \hspace{1cm} (30)

if $D_1 \geq k_{UC1}$.

\textsuperscript{20}This will require $c_2$ large and $\mu$ small enough to make $(1 - \mu)u_2^*(k_H) > \left(\frac{(2\sqrt{c_1(k_H - 1) - k_L})}{k_L}\right)^2$. See more discussion in Appendix C.
It can be shown that as long as $\mu$ is not too large (or alternatively $C_2$ is large, or $F$ is small), then $V_R$ dominates $V_{BD}$ at all $p$.

\[
V_R - V_{BD}(k_L, k_H) = (1 - \mu) \left[ c_2 - \frac{D_2}{k_L} + k_L \right] + (c_1 - D_1) \left[ \frac{(1 - \mu)k_L - 1}{k_L} \right] \\
+ p\theta \left[ D_2 \frac{(1 - \mu)k_L - 1}{k_L} + k_L \frac{(1 - \mu)k_H^2 - k_L}{k_H} \right] - \mu(1 - \mu)F \tag{31}
\]

If $\mu \to 1$, the above expression clearly becomes negative. On the other hand, if $\mu \to 0$, all the terms become positive and as long as $F$ is not very high, $V_R$ will dominate $V_{BD}$ at all $p$. This means that extreme enforcement, which not only causes a small fine $F$ this period, but also eliminates all future returns, will make the official over-extractive, and the outcome will be extremely inefficient.

We assume that $\mu$ is not so high to allow this perverse possibility. In the case of $D_1 \geq k_{UC_1}$, high $F_2$ can render $\tilde{V}_{BD} < 0$ at all $p$, and the official will not find it worthwhile to force both types to default. Therefore we restrict our attention only to strategies $D$ and $R$.

### 3.2 Default or Repay?

The two strategies - ‘both repay’ and ‘only $H$ repays’ - have a tradeoff. If both are to repay, the official must concede a great deal of rent to the $H$ type, but he can expect a large payoff in the second period. On the other hand, should he make $L$ default, his second period payoff will be smaller, but he can extract more from $H$ both now and later.

Given that the strategy $D$ is relatively more attractive in the first period, the scale can tilt in its favor only if $p$ (i.e. the prior on $H$) is high. When the borrower is more likely to be highly productive, it may be optimal to maintain a long-term relation with the high type alone than with both. As does the following proposition show, inducing $L$ to default is indeed optimal at higher values of $p$.

**Proposition 3** (a) There exists a unique $p$, say $p^*$ such that at all $p > p^*$ the corrupt official induces the $L$ type to default. (b) Assuming $V_R$ and $V_D$ both increasing in $p$, if $V'_R(p) < V'_D(p)$ at all $p \leq p^*$ (where $p^*$ is defined in Proposition 2), then $p^*$ is unique, which implies that at all $p \leq p^*$ the official induces repayment by both types. (c) The value of $p^*$ is higher when $D_1 \geq k_{UC_1}$, as compared to when $D_1 < k_{UC_1}$
Proposition 3 establishes our central result. The probability of default by a $L$ type borrower is now unconditionally positive. This can be seen from the fact that $V_D$ exceeds $V_R$ as $p \to 1$, and $V_R$ exceeds $V_D$ as $p \to 0$. Then they must intersect at least once. Suppose $p^*$ is the highest such value of $p$ among the intersection points. In part (b) we then show that if a sufficient condition is met, which is favoured by a small $\mu$ or large $c_2$, the intersection will be unique. This means that repayment (default) by type $L$ will occur only at $p$ below (above) the critical mark $p^*$. Part (c) extends the same result to the case where a $U$ type is excluded through higher penalty. Since default will now invite a sure and higher penalty in the second period, the cutoff probability must rise allowing repayment over a longer range of $p$.

Figure 3 illustrates this case. As is shown, the $V_R$ curve dominates $V_D$ only up to $p^*$. To illustrate how the information rents matter for this decision, we draw the official’s hypothetical payoffs, $W_R$ and $W_D$, had he not given any rents at any $p$, and had there been no distortions in $B$ and $t$ (for each type). We see that $W_R$ dominates $W_D$ over the entire support. While $V_D < W_D$, as is $V_R < W_R$, $V_R$ falls short much more sharply (at high $p$) than $V_D$, giving rise to the above result. Thus, differential screening costs result in the inefficient choice of the mixed contract over the uniform (long term) contract giving rise to inefficiency. The screening cost differs between the two strategies because of both of informational ratchet effect and countervailing incentives. It can be shown that the presence of only one such factors is enough to cause inefficiency.

Note that for the optimality of default, $V_D$ must remain above $V_R$ as $p \to 1$:

$$V_D(1) - \lim_{p \to 1} V_R(1) = c_1 \theta + \theta \frac{D_2 + k_L^2}{k_H} > 0.$$  

Recall from our discussion of equation (11) in Section 3.1.1 that these two terms represent information rents for two periods. Even if we did not have informational ratchet effect, still $V_D$ would dominate $V_R$ by $\theta c_1$ (static rent). That would be enough to make the make strategy $D$ (or the mixed contract) optimal.

---

$W_R = pU_R(k_H) + (1 - p)U_R(k_L)$, and $W_D = pU_R(k_H) + (1 - p)U_D(k_L)$, where $U_R(.)$ and $U_D(.)$ functions are given are Section 2.2. Since by Assumption 1, $U_R > U_D$ for both $H$ and $L$, $W_R \geq W_D$ always holds.
Conversely, if we did not have countervailing incentives, but only dynamic information rent, then also default would appear optimal. We prove this point in Section 7. There is a range of $D_1$, where the reversal of incentive to misrepresent does not occur. For example, at $D_1 \geq \frac{L_c}{k_H} c_1$, it is the $H$ type who has the incentive to misrepresent, regardless of whether the official plays $R$ or $D$. As we show in Section 7, because of the dynamic rent associated with the uniform long term contract, strategy $R$ loses out to strategy $D$, which requires payment of only one period rent.\footnote{Here, it is noteworthy that our argument goes through even if separating equilibrium did not exist under repayment. In that case the official would resort to pooling offers at all or some $p$. But that would have yielded a lower $V_R$ curve, and consequently a smaller $p^*$, which implies even greater inefficiency.}

It is straightforward to calculate the probability of default (in the first period) conditional on $k_i$ being $k_L$, as:

\[
\sigma(p \mid k_L) = \begin{cases} 
0 & \text{for } p \leq p^* \\
(1 - q)(1 - p) & \text{for } p > p^*
\end{cases}
\]

The above analysis lends some support to the view that the long term effect of corruption is much more harmful than its short run incentive effects. We identify a particular type of inefficiency - corruption induced default - that may have a greater social implications. Rose-Ackerman (1999) and many other authors have therefore warned against soft policies on corruption.

To sum up, we have shown that offering a short term contract to a low productivity type can be optimal when the prior on the borrower being high productivity is on the higher side. The key reason for such optimality is the cost of screening the borrower’s type. From a corrupt official’s point of view, offering long term contracts to both types of borrowers may involve yielding dynamic information rents to the high productivity type. But offering a mixed contract (involving a short term contract to the low type and a long term contract to the high type) will eliminate dynamic rents, and further may even allow pegging the high type to his minimum payoff. This is why the mixed contract is optimal, when the borrower is more likely to be a high type.
3.2.1 Comparative statics

In light of the Proposition 3, we can say that, $p^* \geq p$ is a new repayment incentive condition appropriate for a corrupt official constrained by asymmetric information, because it restates $V_R(p^*; \alpha) \geq V_D(p^*; \alpha)$, where $\alpha$ is the vector of parameters (notably, $c_1$ and $c_2$).

How does the critical value, $p^*$, change with the loan terms or anti-corruption measures? The answers are unambiguous and interesting. Suppose $D_1 < k_U c_1$, $p^* > p^C$ and it is unique.\textsuperscript{23} Then we can write,

$$V_D(p^*(\alpha)) - V_R(p^*(\alpha)) \equiv 0,$$

where $\alpha = (c_1, c_2, D_1, D_2, F, \mu)$. This leads us to:

$$\frac{\partial p^*}{\partial \alpha} = \left[ \frac{\partial V_R}{\partial \alpha} - \frac{\partial V_D}{\partial \alpha} \right] \frac{1}{V_D'(p^*) - V_R'(p^*)}.$$

Since $V_D'(p^*) > V_R'(p^*)$, the sign of $\frac{\partial p^*}{\partial \alpha}$ is given by the sign of $\left[ \frac{\partial V_R}{\partial \alpha} - \frac{\partial V_D}{\partial \alpha} \right]$.

It is straightforward to check that,

$$\frac{\partial p^*}{\partial c_1} < 0, \quad \frac{\partial p^*}{\partial c_2} > 0, \quad \frac{\partial p^*}{\partial D_1} > 0,$$

$$\frac{\partial p^*}{\partial D_2} < 0, \quad \frac{\partial p^*}{\partial F} < 0, \quad \frac{\partial p^*}{\partial \mu} < 0.$$

An increase in $p^*$ means an improvement in the prospect of repayment. It is not hard to see why the present and future loan terms affect the repayment prospect in opposite ways. The corrupt official induces repayment in order to appropriate the second period surplus $(k c_2 - D_2)$ of the borrower by paying him $k(c_1 - D_1)$, what he could have got by defaulting in the first period, (recall condition (1)), plus some information rent, if necessary. An increase in $c_1$ thus reduces the official’s payoff and makes repayment a less attractive option. Therefore, $p^*$ falls. In contrast, an increase in $c_2$ directly increases the pie that he appropriates in the second period. Hence, its effect of $p^*$ is positive. Analogous reasonings can be applied to $D_1$ and $D_2$. But more interestingly, the effect of stricter anti-corruption measures are seen to be counter-productive, as was seen under symmetric information. A small increase in $F$ or $\mu$, reduces the official’s expected payoff from repayment much more than that from default. Hence, the prospect for default rises.

\textsuperscript{23}The case of $D_1 \geq k_U c_1$ is similar, and signs are identical.
4 The case of \( k_U \)

We now consider the case where \( k_U \) is realized instead of \( k_L \). Since \( k_U \) is strictly less than 1, advancing loan to \( U \) is socially inefficient. This inefficiency can be resolved by setting \( D_1 \geq k_U c_1 \), in which case the only borrower to approach the official must be the \( H \) type. The corrupt official can easily extract full information payoff from the borrower, and will always induce repayment.

But he may not be able to do so, if \( D_1 < k_U c_1 \), because now \( U \) will also take the loan along with \( H \), and the informational uncertainty will come back. In the case of an honest official, as before the borrower receives the loan at no cost, and \( U \) will default. With a corrupt official it remains to be seen whether \( H \) type will also default or not.

Formally, we consider two strategies of the official: ‘both types default’, \( BD \) and only ‘\( U \) defaults’ (strategy \( UD \)). When both types default, the problem reduces to a one period problem, where the borrower’s gross payoff must be at least \( D_1 \). It can be easily seen that type \( H \) will have to be given one period rent to reveal his type, while type \( U \) is pinned down to zero profit. Following the analysis of Section 3.1.1, we derive the official’s expected payoff from strategy \( BD \) as:

\[
V_{BD} = (c_1 - \frac{D_1}{k_U}) + pk_H + (1 - p) \frac{k_U}{(1 + \rho \gamma k_U)} - \mu F
\]

where \( \gamma = \frac{1}{k_U} - \frac{1}{k_H} \).

On the other hand, when the official wishes to make \( H \) repay (while \( U \) defaults), the size of \( D_1 \) matters for which type to be given rent. We consider two scenarios: First, \( D_1 \in [(k_U - k_H)c_1, (1 + k_L - k_H)c_1) \), i.e. when \( D_1 \) is not small. Second, \( D_1 < (1 + k_U - k_H)c_1 < \frac{k_U c_1}{k_H} \).

Note that \( D_1 = 0 \) is a special case in the second scenario. Very poor borrowers may fall in this category. We shall see that the outcomes are quite different between these two cases.

**The case of \( D_1 > \frac{k_U c_1}{k_H} \):** In this case making \( R \) repay will entail transferring rent to him. Therefore, in terms of bribe and red tape it is similar to \( BD \) strategy, but now there is a second period payoff. Thus,

\[
V_{UD} = V_{BD}(k_U, k_H) + p(1 - \mu)u^*_2(k_H).
\]

Obviously, in this case \( H \) will never be induced to default. But that is not so in the next case.

**The case of \( D_1 < (1 + k_U - k_H)c_1 \):** Now the informational advantage switches in favor of \( U \), exactly the same way observed in Section 3.1.2. Consequently, the official’s expected
utility $V_{UD}$ will be identical to (26) and (27) with $k_L$ being replaced by $k_U$, $\theta$ by $\gamma$ and $p^c$ by $p^U$, where $\gamma$ and $p^U$ are redefined in terms of $k_U$. A direct comparison between $V_{BD}$ and $V_{UD}$ gives the following result.

**Proposition 4** Assume $0 < (1 + k_U - k_H) < \frac{k_H}{k_U}$. If $D_1 \geq \frac{k_H}{k_U} c_1$, inducing $H$ to default is never optimal. But if $D_1 < (1 + k_U - k_H)c_1$, there exists a critical $p$, say $\tilde{p}$, such that at all $p < \tilde{p}$, $H$ will be made to default along with $U$. At all $p \geq \tilde{p}$, the $H$ type repays. Assuming $V_{UD}$ increasing, if $V'_{BD}(p) < V'_{UD}(p)$ at all $p \leq p^U$, $\tilde{p}$ is unique.

For proof see Appendix D.

While the above proposition points to the possibility of greater inefficiency as the high productivity type may also fail to repay, the reason for inefficiency is still the same. The corrupt official’s tradeoff between a mixed contract and a uniform contract is critical. Note that here the uniform contract is a menu of short term offers to both types, and thus, there is no room for dynamic information rent. So, for the inefficient treatment of $H$, we must need countervailing incentive. If $D_1$ is high, there are no countervailing incentives, and $H$ is induced to repay at all $p$. Here, the mixed contract wins over the uniform contract. But if $D_1$ is low either because the government is generous, or because the borrower is poor, countervailing incentives arise between the two contracts, and as before the mixed contract wins over the uniform contract at higher values of $p$, which in this case implies ‘efficiency’ (because $H$ reapys). But at smaller values of $p$, the consequence is disastrous. The high productivity borrower also turns defaulter.

The probability of default, conditional on $k_l$ being realized as $k_U$, is:

\[
\sigma(p \mid k_U) = \begin{cases} 
(1 - q) + q(1 - p) & \text{for } p < \tilde{p} \\
(1 - p) & \text{for } p \geq \tilde{p}
\end{cases}
\]

What are the comparative static properties of $\tilde{p}$? Following the same procedure as in the previous section, it can be shown that the sign of $\frac{\partial \tilde{p}}{\partial \alpha}$ is given by the sign of $[\frac{\partial V_{BD}}{\partial \alpha} - \frac{\partial V_{UD}}{\partial \alpha}]$. Again using (32) and the convex segment of $V_{UD}$ it can be shown that,\n
\[
\text{sign of } \frac{\partial \tilde{p}}{\partial \alpha} = - \text{sign of } \frac{\partial p^*}{\partial \alpha}.
\]
The effect of $\alpha$ may appear exactly opposite of what we have seen earlier, but it is actually working in the same direction. For example, an increase in $c_1$ will increase the chance of $H$ defaulting when $k_U$ is realized, and will also increase the chance of $L$ defaulting when $k_L$ is realized. Greater $c_1$ reduces the official’s future payoff comparatively more than it increases his current payoff. Hence, the inefficiency. Formally, of course, the two cutoff probabilities, which are essentially two repayment incentive conditions from a corrupt official’s point of view, will tend to move apart, if $c_1$ is reduced, or $c_2$ is increased. This is important, as we shall see, for the optimal design of the credit program.

5 Optimal credit program

In this section, we would like to model the government’s choice over the elements of $\alpha = (c_1, c_2, D_1, D_2, \mu, F)$ that can influence the outcome of the game by raising the probability of repayment, $p^*(\alpha)$. In attempting such an analysis, we choose the following objective function:

Government’s objective: $Z = \text{Expected payoff of the borrower} - \text{expected subsidy cost} - \text{expected monitoring cost}$.

This is sought to be maximized, subject to a number of constraints that may reflect the strategy of the government, institutional flexibility in tackling corruption and availability of credit.

First consider the outcome that is efficient from the repayment point of view: if a loan is advanced, it is never defaulted. How does the government ensure this outcome?

We first consider the ideal scenario, where the official is known to be honest, all $U$ type borrowers can be excluded by setting $D_1$ equal to $k_Uc_1$, and $D_2$ can be raised to $c_2$ to eliminate the default problem in the second period as well. The credit program optimal to this environment should be seen as the first best program.

The social welfare function $Z$ will consist of the following expected payoff of the borrower:

$$X_1 = p\Pi(k_H) + (1 - p)\beta\Pi(k_L)$$

where $\Pi(\cdot)$-s are given by (4). Note that a $U$ type does not take the loan.
Now consider the subsidy cost. When the loans are repaid, subsidy cost consists of only interest. For the first period it is $rc_1$, which after one period becomes $rc_1(1 + r)$ where $r$ is the government’s opportunity cost of fund. If the borrower defaults, then not only the interest, but also a part of the principal, $(c_1 - D_1) = c_1(1 - k_U)$, is also lost. The second period subsidy is $c_2r$.

The subsidy cost in an honest regime is:

$$Y_1 = \{p + (1 - p)\beta\} [c_1 r (1 + r) + c_2 r]$$

Finally, since there is no corruption, the monitoring cost is zero.

Assuming that $D_1 = k_Uc_1$ and $D_2 = c_2$, the government’s problem can be stated as:

Maximize with respect to $c_1, c_2$:

$$Z = X_1 - Y_1$$

subject to:

$$c_2 \geq \left[ \frac{(1 - k_U)k_L}{(k_L - 1)} \right] c_1 \tag{33}$$

$$c_1 \leq \bar{c}_1$$

$$c_2 \leq \bar{c}_2.$$

The first constraint is a restatement of condition (3), which ensures that $H$ and $L$ will not willfully default. The second and the third constraints are simply the availability constraints.

Note that the social welfare function and the constraints are all linear. Therefore, we are going to get corner solutions. Assuming $\frac{\partial Z}{\partial c_1} > 0$ and $\frac{\partial Z}{\partial c_2} > 0$ on regularity grounds, two possibilities are noted. If $\bar{c}_2 \geq \left[ \frac{(1 - k_U)k_L}{(k_L - 1)} \right] \bar{c}_1$, then $c_1 = \bar{c}_1$ and $c_2 = \bar{c}_2$. Otherwise, $c_1 = \bar{c}_1 \left[ \frac{k_L - 1}{k_L (1 - k_U)} \right]$ (denote it as $c_1(\bar{c}_2)$), and $c_2 = \bar{c}_2$. The basic point is that the government will make the loans as large as possible, as long as they satisfy the no-willful-default constraint. This is our first best credit program.

In the general case, the first best may not be implementable, for two reasons: (1) corruption, (2) difficulty of excluding $U$ type borrowers. If the borrowers are poor, exclusion of $U$ may not be feasible. In that case, an incidence of default cannot be attributed to corruption, and the official cannot be punished accordingly. The wealth constraint of the borrower may further restrict the size of $D_2$. 

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We consider a particular case of very poor borrowers, where \( D_1 = 0 \), but \( D_2 > 0 \). Since \( D_2 \) does not play an important role we set \( D_2 = c_2 \) for simplification.

Now, the government has to choose \((c_1, c_2)\) by paying attention to two more constraints. To ensure that both \( H \) and \( L \) repay when \( k_L \) is realized, \( p^*(\alpha) \) should be set at least equal to \( p \), and to ensure that \( H \) repays when \( k_U \) is realized, \( \tilde{p}(\alpha) \) must not exceed \( p \). These two are the appropriate repayment incentive conditions for a corrupt regime involving all three types of borrowers.

While the above problem looks at an efficient outcome, an inefficient outcome involving default by \( L \) can also be considered. This will require violating the constraint \( p^*(\cdot) \geq p \). We would like to compare the designs of the credit programs for these two different outcomes, and see if ever the inefficient outcome is preferred.

In order to state the problem formally, we need to specify the monitoring cost. It is not unreasonable to suppose that the monitoring operation is self-financing. Suppose the government chooses \( \mu \) such that \( M = 0 \).\(^{24}\) To simplify further, we assume that \( F \) and \( F_2 \) are exogenously given.\(^{25}\)

Let \( X_2 \) denote the borrower’s expected payoff under corruption and \( Y_2 \) the expected subsidy cost under corruption. Since these expressions are lengthy, we relegate them to Appendix \( E \). The government’s problem is to choose \((c_1, c_2)\), which maximizes

\[
Z = qX_1 + (1 - q)X_2 - qY_1 - (1 - q)Y_2
\]

subject to the following constraints:

\[
p^*(c_1, c_2, D_2(c_2)) \geq p \tag{34}
\]

\[
\tilde{p}(c_1, c_2, D_2(c_2)) \leq p \tag{35}
\]

\[
c_1 \leq \bar{c}_1
\]

\[
c_2 \leq \bar{c}_2.
\]

\(^{24}\)Suppose, the investigating agency, if called to investigate, is paid an exogenous fee \( M \). When not called, it gets nothing. From investigation the government’s total expected revenue is \( \mu(1 - q)F + (1 - \mu)(1 - q)F = (1 - q)F \), but it must pay \( \mu[M + (1 - q)M] + (1 - \mu)M \). Thus, the expected cost equals \( M[1 + \mu(1 - q)] \). Setting expected cost equal to expected revenue, we get \( \mu = \frac{F}{M} \frac{1}{1 - \mu} \). For \( \mu > 0 \), \( F \) must exceed \( M \).

\(^{25}\)\( F \) is likely to be related to the salary of the official. \( F_2 \) can be severe in the sense of being transferred to a different location, or it can be a jail term also.

33
Note that the no-willful default constraint (33) is now replaced by two probability constraints. The first one is to ensure that \( L \) does not default and the second one will ensure that \( H \) does not default when \( U \) is present. As we know these two constraints are much stronger than the no-willful default constraint (compare (3) and (6)). Under corruption (6) must be satisfied, and therefore, the no-willful default constraint is automatically satisfied.

First consider the repayment outcome. Setting the Lagrangian appropriately and assuming that the second order conditions hold, we state the key first order conditions:

\[
c_1: \quad \frac{\partial Z}{\partial c_1} + \lambda_1 \frac{\partial p^*}{\partial c_1} - \lambda_2 \frac{\partial \tilde{p}}{\partial c_1} - \lambda_3 = 0
\]

\[
c_2: \quad \frac{\partial Z}{\partial c_2} + \lambda_1 \left[ \frac{\partial p^*}{\partial c_2} + \frac{\partial p^*}{\partial D_2} \frac{\partial D_2}{\partial c_2} \right] - \lambda_2 \left[ \frac{\partial \tilde{p}}{\partial c_2} + \frac{\partial \tilde{p}}{\partial D_2} \frac{\partial D_2}{\partial c_2} \right] - \lambda_4 = 0
\]

where \( \lambda_i, (i = 1, 2, ..., 4) \) are the Lagrange multipliers for the \( i \)-th constraint. Other first order Kuhn-Tucker conditions for the \( \lambda \)-s are omitted.

While the formal solution is given in Appendix E, here we present an informal discussion. We first ask: Is the first best solution ever admissible? The answer is ‘No’. If it were true, then \( L \) can never repay when the official is corrupt.

The optimal solution hinges on the requirement that \( \tilde{p} \leq p \leq p^* \) must be maintained in equilibrium, and for that reason \( c_1 \) and possibly \( c_2 \) will have to be distorted. Here, the effects of \( c_1 \) and \( c_2 \) on \( \tilde{p} \) and \( p^* \) are important. We know from our discussions in sections 3.2.1 and 4 that \( c_1 \) negatively affects \( p^* \), but positively affects \( \tilde{p} \). It can also be shown that the total effects of \( c_2 \) - direct and indirect (via \( D_2 \) - on \( \tilde{p} \) are negative, but ambiguous for \( p^* \).

Space for Figure 4

These signs play a crucial role and ensure that in general only one of the two probability constraints will bind. Consider a very low value of \( p \). To hold \( \tilde{p} \) below \( p \), \( c_1 \) is to be reduced, while \( c_2 \) is to be raised. Thus, \( c_2 \) is to be held at \( \bar{c}_2 \), and \( c_1 \) is to be adjusted to set \( \tilde{p} = p \). This also helps to keep \( \tilde{p} < p^* \). Clearly, to maintain \( \tilde{p} = p \) with lower \( p \), \( c_1 \) must fall and symmetrically with higher \( p \), \( c_1 \) will rise. However, as shown in Figure 3, beyond a point where
the gap between $\tilde{p}$ and $p^*$ disappears, $c_1$ cannot be increased. If $c_1$ is raised further $p^*$ will fall below $p$, and $L$ will be made to default. Suppose $c_1$ is such that $\tilde{p}(c_1, \bar{c}_2) = p^*(c_1, \bar{c}_2) = p^0$. Then at all $p > p^0$, $c_1$ must be decreasing in $p$ to maintain the equality $p^*(.) = p$. But here $c_2$ may not necessarily be held at $\bar{c}_2$, because the total effect of $c_2$ on $p$ turns negative at some $p > p^\mu$. Suppose $p^\mu > p^0$. Then beyond $p^\mu$ we arrive at an interior solution of the following kind:

$$\frac{\partial Z}{\partial c_1} = \frac{[\partial p^*(c_1) + \partial p^*_{\bar{c}_2} kU]}{[\partial p^*_{\bar{c}_2} + \partial p^*_{D_2}]}$$

(38)

This is a familiar tangency solution when the government is maximizing a linear (expected) social welfare function subject to a quasi-concave iso-probability function $p^*(.)$ which is set equal to $p$. Optimal $(c^*_1, c^*_2)$ will lie within their bounds. Consequently, the difference between the two credit amounts may get smaller.

But how far is repayment possible? It can be checked that $p^*(.)$ will not go to 1, even if $c_1 \to 0$. We have noted that $\lim_{p \to 1} [V_D - V_R] = \theta \left[ c_1 + \frac{D_2 + k_2^2}{k_H} \right]$, does not go to zero if we let $c_1$ go to zero. Suppose $p^*$ goes to some limiting value $p^+ < 1$. Then beyond $p^+$ the repayment strategy is not feasible.

In Figure 4, we draw optimal $c_1$ against $p$. Suppose $\tilde{p}(c_1) = p$ and $p^*(c_1) = p$ can be inverted to write $\tilde{c}_1(p)$ and $c^*_1(p)$ respectively. The two curves move in exactly opposite directions. In order to maintain $p^* \geq \tilde{p}$, $c_1$ cannot be raised beyond $c_0^1$. This gives rise to an inverted U-shaped $c_1$-curve. Optimal $c_2$ is held constant at $\bar{c}_2$ at all $p$ up to $p^\mu$, but afterwards $c_2$ also falls.

How will the social welfare function look like in this case? Figure 4 shows that $Z_R$ should be increasing in $p$ up to $p^0$ and then should decrease, as the credit amounts fall.

Next, we consider the inefficient outcome where the government may permit $L$ type to default, which occurs if $p^*(.) < p$. This effectively requires violating the first constraint. So if the government continues along the $\tilde{c}_1(p)$ curve with $c_2 = \bar{c}_2$, the $L$ type would default at all $p > p^0$. Therefore, when default by $L$ is permitted, the optimal credit program is simply given by $(\tilde{c}_1(p), \bar{c}_2)$. Since $c_1$ is increasing, $Z_D$ will also be an increasing function; but because of default, $Z_D$ will start from a much lower value than $Z_R$ at $p^0$. Eventually $Z_D$ crosses $Z_R$ at a point like $\hat{p}$ as shown in Figure 5. Therefore, permitting default will be optimal at all $p > \hat{p}$. If $Z_D$ and $Z_R$ do not intersect, then $\hat{p} = p^+$. 

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Proposition 5 There is a critical $p$, say $\hat{p}$ such that at all $p > \hat{p}$, it is optimal for the government to permit default, but maintain an increasing profile of $c_1$ while $c_2$ is held at its maximum. But at all $p \leq \hat{p}$ it will adopt a default-proof credit program, but at the cost of reducing $c_1$ to a substantially low level.

Finally, it may be asked how the design of the optimal credit program would change if the government could adjust $D_1$ freely to keep the $U$ type out of contention. If $D_1$ could be set equal to $k_Uc_1$, then constraint (35) would be irrelevant, and $H$ will always repay. To ensure that $L$ repays, $c_1$ must be given by $c_1^*(p)$, which is all through declining. Consequently, $Z_R$ will also be declining. The default strategy on the other hand requires setting a constant $c_1$, which should be equal to $\bar{c}_1(p = 0)$ at all $p$. Such a $c_1$ will permit repayment by $L$ only if the borrower is surely the low type. Because of high and constant $c_1$ and $c_2$, $Z_D$ will be increasing. But now the penalty is severe ($F_2$). Therefore $\hat{p}$ will be greater, but still strictly less that 1. Thus, even if $U$ could be excluded, the problem of default cannot be fully eliminated.

Thus, two aspects appear to be integral. First, though the government can significantly improve the repayment prospect by appropriately designing the loan sequence, the default problem cannot be fully eliminated. In fact the probability that it will settle for an inefficient outcome (by tolerating default) is strictly positive. Second, under-provision of the first period credit seems a natural way to reduce the default probability. Here, we must note that the divergence from the first best level of credit provision occurs due to two reasons: corruption and asymmetric information. If there was no informational uncertainty, then $c_1$ still needs to be reduced below the first best level, but repayment can always be ensured. But asymmetric information introduces an additional source of inefficiency, and beliefs about the borrower’s types begin to play a critical role. Ensuring repayment requires further under-provisioning of the first period credit.
6 Extension I: Moderate $D_1$ - the case of countervailing incentives

In this section we consider some interesting extensions. An increase in $D_1$, collateral or penalty on default, may not necessarily improve efficiency, unless $U$ is excluded. As we show higher $D_1$ can change the default strategy in an interesting way; the official can deny the agent of any information rent, as the agents face countervailing incentives.

Here we drop Assumption 3, and let $D_1$ be greater than $c_1(1 + k_L - k_H)$. However, this generalization affects only the strategy of default (i.e. only $L$ defaults). The ‘both repay strategy’ does not change.

To understand the role of Assumptions 3 let us recall the basic feasibility constraints - repayment constraint for $H$ and individual rationality constraint for $L$ from the problem $P'$:

$$
(\text{RC}_H): \quad t^H + c_1 \leq k_H(c_1 - B^H) \\
(\text{IR}_L): \quad t^L + D_1 \leq k_L(c_1 - B^L)
$$

It can be readily checked that if $D_1 < c_1(1 + k_L - k_H)$ and $D < c_1 \frac{k_L}{k_H}$, the $\text{IR}_L$ curve will lie above the $\text{RC}_H$ curve on the $(t, B)$ plane. That is, the feasible set of offers for $L$ will be larger than the feasible set of offers for $H$. This implies that $L$ will have a systematic incentive to misrepresent and this was ensured by Assumption 3, since $\frac{k_L}{k_H} > (1 + k_L - k_H)$.

If $D_1 > (1 + k_L - k_H)c_1$, no longer will $\text{RC}_H$ lie below $\text{IR}_L$. Instead, the two curves will intersect and then we may suspect that $L$’s incentive to misrepresent will not be uniform (or systematic). Indeed with changes in $D_1$, the informational advantage gradually moves away from $L$ to $H$ revealing a pattern of countervailing incentives.

As is typically the case in such models, the two constraint ($\text{RC}_H$ and $\text{IR}_L$) intersects, and their intersection point will vary with $D_1$ (as we hold other parameters unchanged). In Figure 6, two intersecting constraints are drawn. Let the intersection point $E$ be given by $(\hat{t}, \hat{B})$, where it is straight-forward to check:

$$
\hat{t} = \left( c_1 \frac{k_L}{k_H} - D_1 \right) \frac{k_H}{(k_H - k_L)},
$$

Rewrite $(1 + k_L - k_H) \leq \frac{k_L}{k_H}$ as $k_H^2 - k_H(1 + k_L) + k_L \geq 0$. Setting it as equality, we can show that the resulting quadratic equation has two real roots: 1 and $k_L$. So this inequality is violated if $k_H \in (1, k_L)$. But by assumption $k_H > k_L$. So, $\frac{k_L}{k_H}$ is always greater than $(1 + k_L - k_H)$.  

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\[ \hat{B} = \frac{D_1 - c_1(1 + k_L - k_H)}{(k_H - k_L)}. \]  

(40)

Note that the intersection point varies with \( D_1 \), and it is strictly in the interior if \( D_1 \in (c_1(1 + k_L - k_H), c_1 \frac{k_L}{k_H}) \). In the ensuing analysis the relative position of the full information offers (in comparison to point \( E \)) is going to be crucial.

Insert Figures 6 and 7 here.

For instance, if the full information solutions for \( H \) and \( L \) are given by points \( A \) and \( B \), then we will have a different incentive regime than if they were at points \( C \) and \( F \). By incentive regime we mean which type has the incentive to misrepresent had they been given the full information offers for the same strategy of the official.\(^{29}\) How the point \( E \) will shift vis-a-vis the full information offers depends on the size of \( D \). Since full information \( t^{H*} = k^2_H \) and \( t^{L*} = k^2_L \) are independent of \( D_1 \), by comparing them with \( \hat{t} \) we can easily identify the critical values of \( D_1 \), which give rise to three incentive regimes.

Define \( D_1^{*} \) such that for all \( D_1 > D_1^{*} \), the full information red tape \( t^{H*} \) is greater than \( \hat{t} \). \( D_1^{*} \) is obtained by setting \( \hat{t} = t^{H*} \):

\[ D_1^{*} = c_1 \frac{k_L}{k_H} - k_H(k_H - k_L). \]  

(41)

Similarly, define \( D_1^{**} \) such that for all \( D_1 > D_1^{**} \), the full information red tape \( t^{L*} \) is greater than \( \hat{t} \). \( D_1^{**} \) solves \( \hat{t} = t^{L*} \). It can be readily checked from the following expression as well as from Figure 7 that \( D_1^{**} > D_1^{*} \).

\[ D_1^{**} = c_1 \frac{k_L}{k_H} - k_L(k_H - k_L). \]  

(42)

Figure 7 shows how these critical values of \( D_1 \), which help to determine the relative positions of the full information offers vis-a-vis \( (\hat{t}, \hat{B}) \). As \( D_1 \) starts rising towards \( D_1^{*} \), the gap between \( t^{H*} \) and \( \hat{t} \) narrows, but \( \hat{t} \) still remains higher. This will correspond to offers given by points \( A \) and \( B \) in Figure 6, such that the configuration becomes \( t^{L*} < t^{H*} < \hat{t} \). Once this is understood, it becomes obvious that in this situation the low type will have incentive to misrepresent and the full information offers must be distorted to induce truth telling.

\(^{29}\)For \( L \) it is not exactly the full information offers, but the offers under full information corresponding to the default strategy.
Similarly, when $D_1^* < D_1 < D_1^{**}$, the configuration becomes $t^{L*} < \hat{t} < t^{H*}$, which may correspond to offers given by points like $A$ and $C$ in Figure 6. Here, interestingly the full information offers are incentive compatible. Thus, with the help of Figures 6 and 7 we can identify the following incentive regimes.

1. Regime 1: Type $L$ has incentive to misrepresent if $D_1 < D_1^*$. This is just a continuation of the same incentive regime that we have analyzed in section 3.2. Therefore, the optimal offers will be same as in Proposition 2 except for the corner solution, which is now modified as $(\hat{t}, \hat{B})$ and the critical value of $p^c$ should also be modified accordingly. But what is most interesting is that when the corner solution occurs (i.e. at $(\hat{t}, \hat{B})$), both the $RC_H$ and $IR_L$ bind simultaneously, meaning that the first period net payoffs of both types will be zero.

2. Regime 2: None has incentives to misrepresent if $D_1 \in [D_1^*, D_1^{**}]$. In this case, the official will make the full information offers. Moreover, $H$’s offer will lie outside the $L$’s feasible set, and $L$’s offer will be outside $H$’s feasible set, as shown in Figure 6 by points $A$ and $C$. So there is no incentive problem here.

3. Regime 3: If $D_1 > D_1^{**}$, type $H$ has incentive to misrepresent. With even greater value of $D_1$, $\hat{t}$ will fall below $t_L^*$ and $L$’s offer will be inside $H$’s feasible set. However, while $L$’s red tape will be reduced to lower the information rent, it will never be less than $\hat{t}$, and if $t_L$ hits $\hat{t}$, then $H$ will also be pushed to his repayment constraint.

We make these regimes precise in the following proposition.

**Proposition 6**  
1. Suppose $D_1 \in [c_1(1 + k_L - k_H), D_1^*)$. (a) For $p > p^c = \frac{\sqrt{\tilde{t} k_H}}{\sqrt{\tilde{t} k_H} - k_L}$, the optimal offers are same as in equations (20)-(23). (b) For $p \leq p^c$, $t^H = \tilde{t}$, $B^H = \hat{B}$, $t_L = t^{L*}$ and $B_L = B^{L*}$, resulting in $R_H^1 = c_1$ and $R_L^1 = D$.

2. When $D \in [D_1^*, D_1^{**}]$, optimal offers are: $t^i = k^2_i, i = H, L$ and $B^H = c_1 - k_H - \frac{c_1}{k_H}, B_L = c_1 - k_L - \frac{D}{k_L}, \forall p \in [0, 1]$.

3. Assume $D_1 \in (D_1^{**}, c_1\frac{k_L}{k_H})$. (a) For $p < p^{''} = \left(\frac{k_L - \sqrt{\tilde{t} k_H}}{k_H - \sqrt{\tilde{t}}}\right)\frac{k_H}{k_L}$, the optimal $t^H, t^L$ are same as in Proposition 1 (equations (12) and (14) respectively), and since the low type
defaults, optimal $B^H, B^L$ are given by:

$$
\tilde{B}^H = \left[c_1 \left(\frac{k_H - 1}{k_H} - k_H \right) - \left[\theta(c_1 + \frac{k_L^2}{(1 + \rho\theta k_L)^2})\right]\right] - \left[\theta(c_1 + \frac{k_L^2}{(1 + \rho\theta k_L)^2})\right] \quad (43)
$$

$$
\tilde{B}_L = c_1 - \frac{k_L}{(1 + \rho\theta k_L)^2} - \frac{D_1}{k_L} \quad (44)
$$

(b) For $p \geq p''$, $t^L = \hat{t}$, $B^L = \tilde{B}$, $t^H = t^{H*}$, and $B^H = B^{H*}$. In this case, $R^H_1 = c_1$ and $R^L_1 = D_1$.

4. If $D_1 \in (c_1 \frac{k_L}{k_H}, c_1]$, the optimal offers are same as in part (3.a) for all $p$.

For proof see Appendix F.

The proposition shows how the red tape and bribe will vary with changes in the exogenous penalty and the official’s priors. More appropriately, the default strategy is characterized by several incentive regimes. For a more useful discussion, we can fix $p$ and examine the interior solution as it changes with $D_1$.

At low values of $D_1$, the red tape for $H$ is given by equation (21) or $t^H_A$ as shown in Figure 8, and the red tape for $L$ is at the full information level $t^{L*}$. As $D_1$ reaches $\hat{D}_1$, which is given by the equality of $t^H$ as in (21) and $\hat{t}$, the red tape for $H$ will be restricted at $\hat{t}$ and will continue to be so till $D_1^*$. Particularly in this phase, the red tape is clearly sensitive to the penalty. After this the red tape again becomes insensitive to $D_1$, but it settles down at a lower level at $t^{H*}$. Similarly the red tape of the low type can be understood.

Thus, it appears that when the official prefers to play the default strategy (i.e. only $L$ defaults), the penalty on default reduces the expected delay or red tape in an indirect manner, primarily through regime changes. As the penalty increases from low to moderate and to higher levels, red tape eventually falls for both types - from $t^H_A$ to $\hat{t}$ and then onto $t^{H*}$ for $H$, and from $t^{L*}$ to $t^L_A$ via $\hat{t}$ for $L$. This can be described as an efficiency effect of the policy of increasing penalty on default.

However, a related question is: does it increase profit for the borrower? This relates to information rent accumulation or rent dissipation. As is well known, countervailing incentive models typically exhibit a phase of rent dissipation, and our model is no exception. The only difference in our case, as compared to Lewis and Sappington (1989) and Maggi and
Rodriguez-Clare (1995), is that the rent dissipation is triggered by a change in an exogenous variable. This is similar to Saha (2001). In Proposition 6, we have shown how the first period net profits (or information rent) of the borrower will change. The rent will completely disappear in regime 2, but will come back in regime 3.

To emphasize on the rent dissipation process, we state it more formally in our next proposition, and also provide a visual illustration in Figure 8. The information rent of the low type, which is given in Proposition 2 is declining in $D_1$. But it is strictly positive until $D_1$ reaches $\hat{D}_1$. Thereafter it is zero. It is noteworthy that for all $D_1 < D_1^*$, the low type has incentive to misrepresent (Regime 1). But this incentive is not uniformly strong or profitable. It weakens gradually. In fact, the incentive becomes completely useless well before $D_1^*$ at $\hat{D}_1$.

Similarly, the high type has incentive to misrepresent in Regime 3, at $D_1 > D_1^{**}$. But it does not translate into rent immediately at $D_1^{**}$. Only from $\tilde{D}_1$ the high type begins to enjoy rent, which however increases with $D_1$. Thus, the entire interval $[\hat{D}_1, \tilde{D}_1]$, which includes Regime 2 well inside, is marked by complete dissipation of information rents.\footnote{Recall from Proposition 2, $\pi_1^L = B^H(k_H - k_L) + [c_1(1 + k_L - k_H - D_1)]$ and with the help of Proposition 1 we can derive $\pi_1^H = [c_1 - \tilde{B}^H](k_H - k_L)$ where $\tilde{B}^H$ as shown in equation (44) is inversely related to $D_1$. Therefore, $\pi_1^L$ is inversely and $\pi_1^H$ is directly related to $D_1$.}

**Proposition 7** For a given $p$ there exists an interval of $D_1$, say $[\hat{D}_1, \tilde{D}_1]$, where $c_1(1 + k_L - k_H) < \hat{D}_1 < D_1^*$ and $D_1^{**} < \tilde{D}_1 < c_1 \frac{k_L}{k_H}$, such that at all $D_1 \in [\hat{D}_1, \tilde{D}_1]$, the first period net profit ($\pi_1$) is zero for both types of the borrower. Moreover, $\pi_1^L > 0$, for $D_1 < \hat{D}_1$ and $\pi_1^L = 0$, $\forall D_1 \geq \hat{D}_1$. Similarly $\pi_1^H > 0$ for $D_1 > \hat{D}_1$ and $\pi_1^H = 0$, $\forall D_1 \leq \hat{D}_1$. Formally,

$$\hat{D}_1 = c_1 \frac{k_L}{k_H} - k_H \left( \frac{\rho}{\rho - \theta k_H} \right)^2 (k_H - k_L)$$ \hspace{1cm} (45)

$$\tilde{D}_1 = c_1 \frac{k_L}{k_H} - \left( \frac{k_L}{(1 + \rho \theta)} \right)^2 \left( k_H - k_L \right)^2 (k_H - k_L) \frac{k_L}{k_H}. \hspace{1cm} (46)$$

The proof of this proposition is straight forward and therefore omitted.\footnote{It is easy to check that $\hat{D}_1 > \tilde{D}_1$. Consider the second term in each equation. The second term in (45) is inverse related to $D_1$.} It is now clear that the default strategy is very attractive when the penalty is moderate, as it involves very...
little information rent or none at all. Although we do not carry out a formal comparison with the repayment strategy, it can be speculated that with moderate $D_1$ the official’s incentive to inflict default will increase. In simpler terms, in Figure 3 the $V_D$ curve will be at a much higher level than if $D_1$ was small. Clearly, the range of $p$ for which the outcome will be inefficient (or default-prone) will now be greater.

7 Extension II: High $D_1$ - the ratchet effect alone causing default

Here we consider the case of $D_1 > \frac{k_H}{k_H}c_1 > (1 + k_L - k_H)c_1$. This is an interesting case where the default strategy does not involve countervailing incentives. It is the $H$ type, who will enjoy the informational rent as in the case of strategy $R$. The moot point of this exercise is to show that even when there are no countervailing incentives, ratchet effect (or dynamic rent) can render the repayment strategy suboptimal.

The official’s problem is:

$$Max \ V = p(B^H + 2\sqrt{t^H}) + (1 - p)(B^L + 2\sqrt{t^L}) + (1 - \mu)pu^*_2(k_H) - \mu F$$

subject to

$$(IC_H) : k_H B^H + t^H \leq k_H B^L + t^L$$

$$(IC_L) : k_L B^L + t^L \leq k_L B^H + t^H$$

$$(RC_H) : k_H B^H + t^H + c_1 \leq k_H c_1$$

$$(RC_L) : k_L B^L + t^L + D_1 \leq k_L c_1$$

The official’s offers are as follows:

$$B^H = \left[ c_1 \left( \frac{k_H - 1}{k_H} - k_H \right) \right] - \left[ \theta(c_1 + \frac{k_L^2}{(1 + \rho \theta k_L)^2}) \right]$$

$$t^H = k_H^2$$

$$B^L = \left( \frac{c_1 k_L - D_1}{k_L} \right) - \frac{k_L}{(1 + \rho \theta k_L)^2}$$

$$t^L = \frac{k_L^2}{(1 + \rho \theta k_L)^2}.$$

simply $t_A^L (\frac{k_H}{k_H} - \frac{k_L}{k_L})$, where $t_A^L$ is given by equation (21). The second term in equation (46) is $t_A^L (\frac{k_H}{k_H} - \frac{k_L}{k_L})$, where $t_A^L$ is given by (14). Since $t_A^L < t^H, D_1 < D_1$.  

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The official’s (undiscounted) two-period expected utility is:

\[
V_D = p \left[ \frac{c_1(k_L - 1)}{k_L} \right] + (1 - p) \left[ \frac{(c_1k_L - D_1)}{k_L} \right] + p(1 - \mu)u_2^*(k_H) + \left[ pk_H + (1 - p) \frac{k_L}{1 + \rho \theta k_L} \right] - \mu F
\]

As \( p \to 1 \),

\[
V_D = \frac{c_1(k_L - 1)}{k_L} + k_H + (1 - \mu)u_2^*(k_H) - \mu F,
\]

and as \( p \to 0 \),

\[
V_D = \frac{(c_1k_L - D_1)}{k_L} + k_L + (1 - \mu)u_2^*(k_L) - \mu F.
\]

In contrast, recall \( V_R \), as given in (17). As \( p \to 1 \),

\[
V_R = \frac{c_1(k_L - 1)}{k_L} + k_H + (1 - \mu)u_2^*(k_H) - \frac{D_2 + k_L^2}{k_H} - \mu F,
\]

and as \( p \to 0 \), we have

\[
V_R = \frac{c_1(k_L - 1)}{k_L} + k_L + (1 - \mu)u_2^*(k_L) - \mu F.
\]

Comparing the above expressions, we see that \( \lim_{p \to 1} V_D > \lim_{p \to 1} V_R \) unambiguously. On the other hand, \( \lim_{p \to 0} V_R > \lim_{p \to 0} V_D \) if \( u_2^*(k_L) > \frac{(c_1 - D_1)}{(1 - \mu)k_L} \), which is precisely our Assumption 1. Hence this is also true. Then we can argue that these two curves must intersect at least once. That is enough to prove that at some \( p \) default will be preferred.

Note that the key part of our argument, \( V_R(1) < V_D(1) \), is entirely due to the second period rent. Thus, it is the ratchet effect alone that is causing inefficiency.

**Proposition 8** When \( \frac{k_L}{k_H}c_1 < D_1 < c_1 \), there will be no reversal of incentive to misrepresent on the borrower’s part, as the official switches from repayment to default strategy. Still the ratchet effect alone makes default optimal at higher values of \( p \).

**8 Conclusion**

In this paper we have presented a dynamic model of credit provision to examine how asymmetric information exacerbates inefficiency generated by corruption. A perfectly productive borrower may be induced by a corrupt official to default, an outcome, which the official would not prefer under symmetric information. The government can reduce this inefficiency, but
will have to reduce the supply of credit. Two factors in our model, informational ratchet effect and countervailing incentives, which are commonly associated with many agency relationships, are acting jointly or alone to cause default and in turn under-provisioning of credit. The countervailing incentive problem can be avoided by placing a substantial collateral requirement. But in the context of offering subsidized credit to poor borrowers, this may not be feasible. Informational ratchet effect, however, is unavoidable as long as the borrower’s productivity is unchanged over time. Thus, there is no clear policy choice for the government, except that corruption needs to be dealt with much more firmly. If the official is honest, or if honesty can be induced by harsh anti-corruption measures, asymmetric information would hardly matter. The first best credit program can be implemented.

Now in light of our model, we would like to comment on some of the empirical findings. As said at the outset, our analysis concerns those cases where the loan official has substantial bargaining power and the borrowers are at his mercy. Default in such cases is common, and to what extent corruption can cause it, can be understood with our model. But there is another group of defaulters, who by the use of their political power and wealth, force the loan official to divert credit to them. Can our model explain this type of default? Since such borrowers are likely to have greater bargaining powers, the honesty of the official does not matter. Their default decisions can be explained to some extent by the basic no-willful-default condition (3), with two caveats. First, a powerful borrower may influence the official to weaken the penalty $D_1$ applicable to him. This can upset the inequality in favor of default. Second, this borrower may have multiple borrowing opportunities, unlike poor borrowers. Default in one bank may not deter him from future loans offered by another bank.

We have also assumed that the official remains in office for two periods. However, transferring officials from one place to another is a common practice. Effect of tenure stability on corruption is an important issue. Choi and Thum (2003) examined how tenure stability of a corrupt official affects corruption. In our model, if the official knows that he may be transferred with some probability, then the long-term contract loses its attractiveness, and the short-term contract will be preferred. In other words, transfer policy will generate greater inefficiency. However, we must add one qualification. In our model, the official knows that the borrower cannot influence the probability of investigation by reporting corruption. Suppose that we drop this assumption and allow the borrowers to vary in terms of their ability
to influence $\mu$ (or provide hard evidence). Shorter duration of tenure in that environment translates into a greater uncertainty about $\mu$. Consequently, the official may be compelled to be less extractive in order to generate a lower probability of investigation.

Finally, we did not consider production uncertainty, which is a common assumption in a credit model. Introduction of this assumption would certainly make our model more general. We still show that informational uncertainty can lead to a situation where the borrower defaults, because he is rendered ‘unable to repay’, and not because he is ‘unwilling to repay’.

Appendix A

Proof of Proposition 1. (a) Assume that separating equilibrium exists. Then it can be shown that of the four constraints only $IC_H$ and $RC_L$ will bind. This proof is standard, and therefore omitted. Substitute the expressions for $B_L$ and $B_H$ obtained from $IC_H$ and $RC_L$ into the official’s objective function, and then carrying out maximization, obtain the first order conditions (11)-(14).

(b) Straight-forward substitution of optimal $(B, t)$ obtained in (a) will yield these profit expressions.

Alternatively, first rewrite the $IC_H$ and $IC_L$ in terms of net (post-repayment) first period profit $\pi_1^H$ and $\pi_1^L$ (by subtracting $c_1$ from both sides, and then assert the following:

$$\left[\pi_1^{H,L} + \theta(D_2 + k_2^L)\right] - \pi_1^{L,L} \leq \pi_1^{H,H} - \pi_1^{L,L} \leq \pi_1^{H,H} - \pi_1^{L,H}$$

(51)

This inequality simply follows from the incentive constraints. As before $\pi_1^{i,j}$ refers to the first period net profit of type $i$ when it misreports as $j$. Substituting $\pi_1^{i,j} = k_i(c_1 - B^j) - t^j - c_1$, $i, j = H, L$, we obtain:

$$\theta(D_2 + k_2^L) + (c_1 - B^L)(k_H - k_L) \leq \pi_1^{H,H} - \pi_1^{L,L} \leq (c_1 - B^H)(k_H - k_L)$$

Since $k_H > k_L$, the above inequality implies $B^L > B^H$. Moreover, the first part of the inequality suggests:

$$\pi_1^{H} \geq \pi_1^{L} + (c_1 - B^L)(k_H - k_L) + \theta(D_2 + k_2^L).$$

(52)

where $\pi_1^{H} \equiv \pi_1^{H,H}$ and $\pi_1^{L} \equiv \pi_1^{L,L}$. The official will maximize his expected utility, by setting $\pi_1^{L} = 0$, and raising $B^L$ and $B^H$ while maintaining $B^L > B^H$, such that the above inequality holds with equality. The rest follows. QED
Appendix B

Proof of Proposition 2. (a) In analogy with the proof of Proposition 1, it can be shown that at the optimum only $RC_H$ and $IC_L$ will bind, and $B^L \geq B^H$ must also hold. Next, substituting $RC_H$ and $IC_L$ into the objective function, and maximizing it we get (18)-(19). However, for $B^H > 0$, we need

$$\frac{c_1(k_H - 1)}{k_H} > k_H(\frac{\rho}{\rho - \theta k_H})^2$$

which gives rise to the restriction: $p > p^c$. For $p \leq p^c$, $B^H$ must be zero. This reasoning gives (18) and (19).

(b) For $p \leq p^c$, clearly $\pi_H^1 = 0$ and $\pi_L^1 = (c_1 - D_1) - c_1(k_H - k_L) = c_1(1 + k_L - k_H) - D_1 > 0$ by assumptions 2 and 3.

For $p > p^c$ we follow the same procedure as in part (b) of Proposition 1, and obtain:

$$\pi_L^1 \geq \pi_H^1 + (c_1 - D_1) - (c_1 - B^H)(k_H - k_L)$$

which can be forced to hold with equality along with $\pi_H^1 = 0$. Next, rearranging the terms we obtain:

$$\pi_L^1 = B^H(k_H - k_L) + [c_1(1 + k_L - k_H) - D_1]$$

which is strictly positive by assumption 3. QED

Appendix C

Proof of Proposition 3. First we establish in several steps that the $V_R$ and $V_D$ curves must intersect at least once, and then we show that given some assumptions, they will intersect only once.

a) Step 1: $V_D(0) < V_R(0)$.

Proof: As $p \to 0$, $V_R = c_1(k_L - 1) + k_L - \mu F + (1 - \mu)u_2^L(k_L)$ where $u_2^L(k_L)$ is given in (7), while $V_D = k_L + c_1(k_H - 1) - \mu F$. Compare the two expressions. Starting with $V_R(0) > V_D(0)$, we get $(c_2 - c_1) > \left[\frac{D_L}{k_L} - D_1 - k_L + \mu F\right] + \frac{c_1(k_H - k_L)}{k_L(1 - \mu)}$. Recall, from Assumption 1, $(c_2 - c_1) > \left[\frac{D_L}{k_L} - D_1 - k_L + \mu F\right] + \frac{c_1-D_1}{k_L(1-\mu)}$. It is easy to check that given Assumption 3, (i.e. $D_1 < (k_L + 1 - k_H)c_1$), $(\frac{c_1-D_1}{k_L(1-\mu)}) > \frac{c_1(k_H-k_L)}{k_L(1-\mu)}$. Hence, $V_D(0) < V_R(0)$. 46
Step 2: $V_D(1) > \lim_{p \to 1} V_R$.

Proof: When $p \to 1$, $V_R \to \bar{V}_R = \frac{c_1(k_L - 1)}{k_L} + k_H + (1 - \mu)u_2^*(k_H) - \theta \frac{D_2 + k_L^2}{k_H} - \mu F$, whereas $V_D(1) = \frac{c_1(k_H - 1)}{k_L} + k_H + (1 - \mu)u_2^*(k_H) - \mu F$. It is now straightforward to see that $V_D(1) - \bar{V}_R = c_1\theta + \theta \frac{D_2 + k_L^2}{k_H} > 0$. Hence at all $p$ sufficiently close to 1, $V_D > V_R$.

Since both $V_D$ and $V_R$ are continuous at all $p \in (0, 1)$, they must intersect at least once.

b) Step 3: If $\mu \leq \frac{k_H - 1}{k_R}$, $V_R$ is increasing at all $p$.

Proof: Differentiating $V_R$ with respect to $p$ we get, 

$$V_R'(p) = k_H \left( \frac{(k_H - k_L)^2}{(k_H - pk_L)^2} + (k_H - k_L) \left( \frac{k_L^2(1 - \mu) - k_L}{k_H^2} \right) + D_2 \theta \left( \frac{k_H(1 - \mu) - 1}{k_H} \right) \right).$$

If $\mu \leq \frac{k_H - 1}{k_R}$, the above expression is positive at all $p$. Moreover, note that $V_R''(p) > 0$.

Step 4: If $c_2$ is sufficiently large, $V_D$ is increasing in $p$.

Proof: Consider $V_D'(p)$. For $p \leq p^c$,

$$V_D'(p) = (1 - \mu)u_2^*(k_H) - \left[ \frac{c_1(k_H - 1)}{k_L} + k_L - 2\sqrt{c_1(k_H - 1)} \right],$$

and for $p > p^c$,

$$V_D'(p) = (1 - \mu)u_2^*(k_H) - k_L \left[ \frac{(k_H - k_L)^2}{(pk_H - (k_H - k_L))^2} \right].$$

However, without restrictions on $\mu$ or $c_2$ the signs of $V_D'(\cdot)$ cannot be ascertained. Assume that $c_2$ is large enough to ensure that.

But note that at $p \leq p^c$, $V_D'(\cdot)$ is constant and at $p > p^c$, it is increasing in $p$. Thus if a sufficient condition can be provided to ensure that the linear segment of $V_D$ is increasing, then the convex segment will also be increasing.

Next, assuming that $V_R$ and $V_D$ both increasing, we suggest a slope condition to ensure uniqueness of $p^c$.

Step 5: If $V_R'(p) < V_D'(p)$ for all $p \leq p^c$, then $p^c$ is unique.

Proof: It is known that $V_D(0) < V_R(0)$ and $\bar{V}_R < V_D(1)$. In addition $V_D$ and $V_R$ are assumed to be increasing. Both are also convex functions. Therefore, if the two curves intersect more than once, it must be on the linear segment of the $V_D$ function. In that case, $V_R$ curve must intersect the linear part of $V_D$ first from above (maintaining a lower slope than $V_D$) and then from below (maintaining a higher slope than $V_D$). If this is ruled out then, we have a unique intersection. Hence, the assumption $V_R'(p) < V_D'(p)$ at $p \leq p^c$. 

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(c) When \( D_1 = k_Uc_1 \), no \( U \) type is expected to take the loan, and hence a default by \( L \) will invite investigation in the next period with certainty. Therefore, \( V_D \) curve shifts down, and \( p^* \) increases. QED

Appendix D

Proof of Proposition 4. Under the strategy of ‘both types default’ the official’s problem is the following:

\[
\text{Max } p(B^H + 2\sqrt{t^H}) + (1-p)(B^L + 2\sqrt{t^U}) - \mu F
\]

subject to

\[
( IC_H ) : \quad k_H B^H + t^H \leq k_H B^U + t^U \\
( IC_U ) : \quad k_U B^U + t^U \leq k_U B^H + t^H \\
( IR_H ) : \quad k_H B^H + t^H + D_1 \leq k_H c_1 \\
( IR_U ) : \quad k_U B^U + t^U + D_1 \leq k_U c_1
\]

It is easy to check that \( IR_U \) and \( IC_H \) will bind resulting in the following offers:

\[
B^H = (c_1 - \frac{D_1}{k_U} - k_H) - \gamma \frac{k_U^2}{(1 + \rho \gamma k_U)^2}, \quad t^H = k_H^2 \\
B^U = (c_1 - \frac{D_1}{k_U} - \frac{k_U}{(1 + \rho \gamma k_U)^2}), \quad t^U = \frac{k_U^2}{(1 + \rho \gamma k_U)^2},
\]

where \( \gamma = k_H - k_U \). Substituting these offers in the utility function of the official, we get \( V_{BD} \) as shown in (32). Further, \( V'_{BD}(p) = k_H \frac{(k_H - k_L)^2}{(k_H - \rho k_U)^2} > 0 \) and \( V''_{BD}(p) > 0 \).

Now, consider the strategy \( UD \). The official’s problem is now:

\[
\text{Max } p(B^H + 2\sqrt{t^H}) + (1-p)(B^L + 2\sqrt{t^U}) - \mu F + p(1-\mu)u_2'(k_H)
\]

subject to \( ( IC_H ), ( IC_U ) \) and \( ( IR_U ) \) as above and

\[
( IR_H ) : \quad k_H B^H + t^H + c_1 \leq k_H c_1.
\]

First the case of \( \frac{k_U}{k_H} c_1 < D_1 < (1 + k_L - k_H) \). Compare the two \( IR \) constraints. As long as \( D_1 > \frac{k_U}{k_H} c_1 \), any offer acceptable to \( U \) is also acceptable to \( H \). Therefore, \( H \) needs to be
given rent to reveal his type. Again, as in the case of strategy $BD$, $IC_H$ and $IR_U$ bind. So the solution remains unchanged, and consequently $V_{UD} = V_{BD} + p(1 - \mu)u^*_2(k_H)$.

Next, $D_1 < (1 + k_U - k_H) < \frac{k_U}{k_H} c_1$. Comparing the two $IR$ constraints, it can be seen that any offer acceptable to $H$ is also acceptable to $U$. Hence, $U$ will receive the rent. Since this is identical to strategy $D$, we can follow the identical procedure and obtain:

For $p \leq p^U = \gamma k_U \left[ \frac{2\sqrt{c_1(k_H - 1)}}{\sqrt{c_1(k_H - 1) - k_U}} \right]$, 

\[ V_{UD} = p \left( 2\sqrt{c_1(k_H - 1)} + (1 - \mu)u^*_2(k_H) \right) + (1 - p) \left( \frac{c_1(k_H - 1)}{k_U} + k_U \right) - \mu F. \] (53)

For $p > p^U$, 

\[ V_{UD} = \frac{c_1(k_H - 1)}{k_H} + p(1 - \mu)u^*_2(k_H) + \left( \frac{p - k_H \rho}{\rho - \gamma k_H} + (1 - p)k_U \right) - \mu F. \] (54)

Now compare $V_{UD}$ with $V_{BD}$, which was given in (32). As $p \to 0$, $V_{BD}(0) = (c_1 - \frac{D_1}{k_U} + k_U - \mu F) > \lim_{p \to 0} V_{UD} = \left( c_1 \frac{(k_H - 1)}{k_U} + k_U - \mu F \right)$ requires $c_1(1 + k_U - k_H) > D_1$, which is precisely the case we are considering.

On the other hand, as $p \to 1$ \lim_{p \to 1} V_{BD} = (c_1 - \frac{D_1}{k_U} + k_H - \mu F), whereas $V_{UD}(1) = \frac{c_1(k_H - 1)}{k_H} + k_H + (1 - \mu)u^*_2(k_H) - \mu F$. Write, as $p \to 1$, 

\[ V_{UD} - V_{BD} = (1 - \mu) \left( u^*_2(k_H) - \frac{1}{(1 - \mu) - \frac{c_1k_U - D_1k_H}{k_H k_U}} \right). \]

From our Assumption 1, $u^*_2(k_H) > \frac{1}{(1 - \mu) - \frac{c_1 - D_1}{k_H}}$. It can be easily checked that $\frac{c_1 - D_1}{k_H} > \frac{c_1k_U - D_1k_H}{k_H k_U}$. Hence, $V_{UD} > V_{BD}$ as $p \to 1$.

Since the two functions are continuous, they will intersect at least once. We also know that $V_{BD}$ is increasing. So assuming $V_{UD}$ also increasing, we can specify the slope condition as in Proposition 3, to ensure uniqueness. QED

Appendix E

Optimal credit program when $D_1 = 0$
First we need to specify the borrower’s payoff. When the official is honest, the borrower receives:

\[ X_1 = p \bar{\Pi}(k_H) + (1 - p) \beta \bar{\Pi}(k_L) + (1 - p)(1 - \beta) \bar{\Pi}(k_U) \]

But when the official is corrupt, his expected payoff becomes:

\[ X_2 = \beta \left[ \Omega_L \{p \Pi^H + (1 - p) \Pi^L\} + (1 - \Omega_L) \{p \Pi^H + (1 - p) \Pi^L\} \right] \\
+ (1 - \beta) \left[ \Omega_U \{p \Pi^H + (1 - p) \Pi^L\} + (1 - \Omega_U) \{p \Pi^H + (1 - p) \Pi^L\} \right], \]

where \( \Omega_L \) is a dummy variable assuming 1 or 0, depending on whether the official induces both \( H \) and \( L \) to repay or forces \( L \) to default, respectively; \( \Omega_L = 1 \) if \( p^*(\alpha) \geq p \), otherwise \( \Omega_L = 0 \). The \( H \)-type borrower’s profit \( \Pi^H \) and \( \Pi^L \) correspond to repayment and default respectively as given in equations (15) and (24). Analogously, \( \Pi^L \) and \( \Pi^L \) are given in (16) and (25). The last term \( \Pi(k_H) \) corresponds to the full information profit of \( H \) as given in (8). Similarly, \( \Omega_U \) is a dummy variable for the state \( (k_H, k_U) \). \( \Omega_U = 1 \), if \( p \geq \bar{p}(\alpha) \), otherwise \( \Omega_U = 0 \).

The subsidy cost is given as:

\[ Y_1 = \{p + (1 - p)\beta\} \left[ c_1 r(1 + r) + c_2 r + (c_2 - D_2) \right] \\
+ (1 - p)(1 - \beta) \left[ (c_1 r(1 + r) + c_1 (1 + r) \right] \]

for an honest regime, and the same for a corrupt regime is:

\[ Y_2 = c_1 r(1 + r) + p \left[(\beta + (1 - \beta)\Omega_U)(c_2 r + (c_2 - D_2) + (1 - \beta)(1 - \Omega_U)c_1 (1 + r)\right] \\
+ (1 - p) \left[ \beta \Omega_U (c_2 r + (c_2 - D_2) + (1 - \Omega_U)(1 + r)c_1 \right] \]

The monitoring cost is zero as before.

The first order conditions of the government’s maximization problem are:

\[ \frac{\partial Z}{\partial c_1} + \lambda_1 \frac{\partial p^*}{\partial c_1} - \lambda_2 \frac{\partial \bar{p}}{\partial c_1} - \lambda_3 = 0 \] (55)

\[ \frac{\partial Z}{\partial c_2} + \lambda_1 \frac{\partial p^*}{\partial c_2} + \lambda_2 \frac{\partial p^*}{\partial D_2} \frac{\partial D_2}{\partial c_2} - \lambda_4 = 0 \] (56)

\[ \lambda_1 \left[ p^*(\alpha) - p \right] = 0 \] (57)

\[ \lambda_2 \left[ p - \bar{p}(\alpha) \right] = 0 \] (58)

\[ \lambda_3 \left[ \bar{c}_1 - c_1 \right] = 0 \] (59)

\[ \lambda_4 \left[ \bar{c}_2 - c_2 \right] = 0. \] (60)
We begin with several observations:

1. First we note that repayment requires setting \( \Omega_L = \Omega_U = 1 \), i.e. ensuring \( \tilde{p} \leq p \leq p^* \).

2. The limiting behaviors of \( \tilde{p}(\cdot) \) and \( p^*(\cdot) \) functions vis-a-vis \( c_1 \) are as follows. For some given \( c_2 \), as \( c_1 \) goes to zero, \( \lim \tilde{p} \to 0 \) and \( \lim p^* \to p^+(c_2) < 1 \). The latter can be proved by checking that with \( c_1 = 0 \), \( \lim_{p \to 1} [V_D - V_R] > 0 \). Hence, \( \lim_{c_1 \to 0} p^*(c_1) \) must be less than 1. Let this limit be called \( p^+(c_2) \).

On the other hand, \( p^*(c_1) \to 0 \), when \( c_1 \to c_1^{++} < c_1(\tilde{c}_2) \), where \( c_1^{++} \) solves \( (1 - \mu)u_2^*(k_L) = c_1^{++} - \mu k_L \), and \( \tilde{p}(c_1) \to 1 \), if \( c_1 \to c_1^+ < c_1(\tilde{c}_2) \), where \( c_1^+ \) solves \( (1 - \mu)u_2^*(k_L) = c_1 \).

3. For \( c_2 \) we know the following. \( [\frac{\partial p^*}{\partial c_2} + \frac{p^*}{\partial D_2} \frac{\partial D_2}{\partial c_2}] \) is positive (nonpositive) if \( p < (\geq)p^\mu \) where \( p^\mu \) is already defined in Section 4. \( [\frac{\partial p}{\partial c_2} + \frac{\partial p}{\partial D_2} \frac{\partial D_2}{\partial c_2}] \) is negative.

4. If \( c_1 = c_1(\tilde{c}_2) \) as in the first best contract, then \( L \) can never pay bribe. This is obvious. By definition \( c_1 = c_1(\tilde{c}_2) \) just satisfies the no-willful-default condition for \( L \) when the official is known to be honest. This allows for no bribe and therefore cannot satisfy condition (6).

From the above observations it follows that at all \( p \leq p^\mu \), an increase in \( c_2 \) will help raising \( p^* \) and reducing \( \tilde{p} \) in favor of repayment. Therefore, \( c_2 = \tilde{c}_2 \) must be optimal, which implies that \( \lambda_4 > 0 \). It also follows that \( c_1 < \tilde{c}_1 \), and hence \( \lambda_3 = 0 \) must hold.

Of the two probability constraints almost always only one will bind. Define \( \tilde{c}_1 \) such that \( \tilde{p}(c_1) = p \) holds, and \( c_1^* \) such that \( p^*(c_1) = p \) holds. Then at a given \( p \), \( \tilde{c}_1 < c_1^* \) if \( \tilde{p} < p^* \), and vice versa. Suppose given \( c_2 = \tilde{c}_2 \), \( \tilde{p} = p^* = p^0 \). Then, at \( p^0 \), \( \tilde{c}_1 = c_1^* \). Further assume that both \( \tilde{c}_1 \) and \( c_1^* \) can be written as continuous functions of \( p \). Now we make the following claim.

**Lemma 1** For repayment, the optimal credit sequence is \((\tilde{c}_1(p), \tilde{c}_2)\) for \(p \in (0, p^0)\), \((c_1^*(p), \tilde{c}_2)\) for \(p \in [p^0, p^\mu]\), and \((c_1^*, c_2^*)\) for \(p \in (p^\mu, p^+)\), where \( c_2^* \) is given by the following tangency condition:

\[
\frac{\partial Z}{\partial c_1} = \frac{[\partial p^*]}{[\partial p^* + \partial p^* \frac{\partial D_2}{\partial c_2}]} \]
Proof: First consider \( p < p^0 \), where \( \tilde{c}_1(p) < c^*_1(p) \). We note that \( \lambda_4 > 0 \) because \( c_2 = \bar{c}_2 \). Suppose \( c_2 < \bar{c}_2 \). Then \( \lambda_4 = 0 \). Then (56) does not hold with equality. As all the terms are positive, \( c_2 \) must be increased. Hence, \( c_2 = \bar{c}_2 \) must hold and \( \lambda_5 \) is determined from equality. Given \( c_2 = \bar{c}_2 \), we show that optimal \( c_1 \) must be given by \( \tilde{c}_1(p) \). For \( \Omega_U = \Omega_L = 1 \), it is necessary that \( p^1(p) \) holds. With \( \tilde{c}_1 \), we get \( p^1(\tilde{c}_1(p)) = p < \bar{p}(\tilde{c}_1(p)) = p \), as long as \( \tilde{c}_1(p) < c^*_1(p) \), which is precisely the case when \( p < p^0 \). Therefore, \( \lambda_1 = 0 \) and \( \lambda_2 > 0 \), which is given by (55).

Next, consider \( p \geq p^0 \) where \( \tilde{c}_1(p) > c^*_1(p) \). By the logic given above, now \( p^* = p \) must hold and, therefore, \( \lambda_1 > 0, \lambda_2 = 0 \) and \( c_1 = c^*_1(p) \) must be the solution. As for \( c_2 \), for \( p < p^\mu \) \( c_2 = \bar{c}_2 \) is optimal, because \( c_2 \) increases \( p^* \). Beyond \( p^\mu \), \( c_2 \) must fall short of \( \bar{c}_2 \), and therefore, \( \lambda_4 = 0 \) and hence, the tangency solution.

Lemma 2 When default by \( L \) is permitted, the optimal credit sequence is \((\tilde{c}_1(p), \bar{c}_2)\).

Proof: Follows from the previous lemma. No longer the constraint (34) is an issue. Hence, \( \tilde{c}_1(p) \) remains optimal at all \( p \), though \( L \) will default only from \( p^0 \) onwards.

Let the optimal social welfare function under repayment be denoted as \( Z^*_R \) and the same, when default by \( L \) is permitted, be denoted as \( Z^*_D \).

Lemma 3 \( Z^*_R \) is increasing at \( p < p^0 \) and then may be decreasing, while \( Z^*_D \) is always increasing at all \( p \in (p^0, 1) \). If \( Z^*_R \) is sufficiently decreasing, then \( Z^*_D \) will intersect \( Z^*_R \) at \( \hat{p} < p^+ \).

Proof: By definition, \( Z^*_R \) is given by the government’s optimal program. Hence, by the envelope theorem we can get:

\[
\frac{\partial Z^*_R}{\partial p} = \frac{\partial Z}{\partial p} + \lambda_2 \quad \text{for} \quad p \in (0, p^0)
\]

\[
= \frac{\partial Z}{\partial p} - \lambda_1 \quad \text{for} \quad p \in [p^0, p^+)\n\]

We must assume that \( \partial Z/\partial p > 0 \) for regularity. Then at all \( p < p^0 \), \( \partial Z^*_R/\partial p > 0 \). For \( p > p^0 \), the sign will depend on the relative magnitudes of the two terms. As \( c_1 \) falls in this region, \( \partial Z/\partial p \) must be declining. Hence the sign can be negative.
On the other hand,
\[ \frac{\partial Z^*_p}{\partial p} = \frac{\partial Z}{\partial p} + \lambda_2 \quad \text{for } p \in (p^0, 1) \]

Clearly it is increasing. The rest of the argument follows. QED

**Appendix F**

**Proof of Proposition 6.**

1. Assume \( D_1 \in [c_1(1 + k_L - k_H), D^*_1] \). Then by checking \( RC_H \) and \( IR_L \) it can be ascertained that full information offers are not incentive compatible; in particular they bind \( RC_H \) but leave \( IR_L \) nonbinding. Then from Proposition 2 it follows that \( IC_L \) and \( RC_H \) must bind, and the offers are given by equations (20) - (23) subject to a restriction that \( t^H < \tilde{t} \). By setting \( t^H \) (as given in equation (21)) equal to \( \tilde{t} \), we obtain the critical value \( p^{c'} \). Since \( t^H \) is inversely related to \( p \) (or \( \rho \)), equations (20) - (23) are valid at all \( p > p^{c'} \).

This completes part (a).

For part (b) we claim that the optimal offers are: \( (t^H = \tilde{t}, B^H = \tilde{B}) \) and \( (t^L = t^{L*}, B^L = B^{L*}) \). Suppose not. Suppose \( (B^H, t^H) \) are given by (20) and (23). Then \( t^H \) must exceed \( \tilde{t} \) and \( B^H \) must be less than \( \tilde{B} \). Since \( RC_H \) binds such an offer is acceptable to \( H \). But since \( t^{L*} < \tilde{t} < t^H \) and \( B^L > \tilde{B} > B^H \), the full information offer for \( L \) will be strictly incentive compatible with \( IC_L \) not binding. Though \( (t^{L*}, B^{L*}) \) will be optimal with respect to \( L \), we know from Proposition 2 (see Appendix B) that optimality requires \( IC_L \) to bind. A closer inspection of problem \( P' \) reveals that if \( t^H \) is reduced and \( B^H \) is increased such that the equality in \( RC_H \) is maintained, the right hand side in \( IC_L \) falls. Since such a movement is toward the first best offer, the official’s payoff from \( L \) must be improving, while for \( H \) it remains unchanged. Since such improvement is possible as long as \( t^H > \tilde{t} \), we have a contradiction. The optimal offer for \( H \) must be \( t^H = \tilde{t} \) and \( B^H = \tilde{B} \). Since these offers bind both \( RC_H \) and \( IR_L \), net profit is zero for both types.

2. Let \( D_1 \in [D^*_1, D^{1*}_1] \). Our claim is that the full information offers as shown in the proposition are optimal. To verify this it can be checked that \( RC_H \) and \( IR_L \) both bind and \( IC_L \) and \( IC_H \) both do not bind. Since the constraints are satisfied and the offers are first best offers, no other offers can do better.
3. Suppose \( D_1 \in (D_{1*}, c_1 k_H/L_H] \). It can be shown that full information offers bind \( IR_L \) but leave \( RC_H \) nonbinding. Therefore, \( H \) will have incentive to misrepresent, which was the case under strategy \( R \). Then by adapting the arguments given in the proof of Proposition 1 (see Appendix A) we can claim that \( IC_H \) and \( IR_L \) will bind, and optimal \( t^H \) and \( t^L \) will be given by equations (12) and (14) respectively. Optimal \( B^H \) is given by \( IC_H \) and optimal \( B^L \) is given by \( IR_L \) which are shown as \( \tilde{B}^H \) and \( \tilde{B}^L \). These offers must observe a restriction that \( t^L > \tilde{t} \). By setting \( t^L \) (as given in equation (14)) equal to \( \tilde{t} \), we obtain the critical value \( p^{c'} \). Since \( t^L \) is also inversely related to \( p \) (or \( \rho \)), these offers are valid if \( p < p^{c'} \). This completes the proof of part (a). The proof of the remaining part is analogous to the first case.

4. When \( D_1 \in (c_1 k_H/L_H, c_1] \), the \( IR_L \) curve will be strictly below \( RC_H \). By the logic used in Proposition 1, only \( IR_L \) and \( IC_H \) will bind as in part (3.a). The rest follows.

QED

References


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Figure 1: Optimal offers when and $H$ and $L$ repay
Figure 2: Optimal offers when L defaults
Figure 3: Optimality of default
Figure 4: Optimal $c_1$ for repayment
Figure 5: The government’s tolerance for default
Figure 6: Intersecting IR constraints
Figure 7: Different incentive regimes
Figure 8: Red tape and penalty on default
Figure 9: Information rents