Delegation and Emission Tax in a Differentiated Oligopoly

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Abstract

This paper examines how product differentiation as well as strategic managerial delegation affects optimal emission tax rate, environmental damage and social welfare, under alternative modes of product market competition. It shows that, under pure profit maximization, the (positive) optimal emission tax rate is not necessarily decreasing in degree of product differentiation, irrespective of the mode of competition. The possibility of emission tax rate to be positive and lower for more differentiated products, under quantity (price) competition, is higher (lower) in case of delegation than that in case of no delegation. It also shows that, under quantity (price) competition, the equilibrium emission tax rate, environmental damage and social welfare are higher (lower) in case of delegation than that in case of no delegation.

Key words: Emission tax, price competition, product differentiation, quantity competition, strategic managerial delegation

JEL Code(s): H23, Q50, Q58, L13
1 Introduction

This paper analyses how optimal emission taxes are affected by the interaction between strategic managerial delegation in firms and degree of product differentiation under alternative modes of product market competition, quantity and price. Whether optimal emission tax rate is higher for more differentiated products, and whether strategic managerial delegation in firms calls for higher emission tax rate to control pollution are essential concerns. Our framework allows us to address the question of how the mode of product market competition affects these comparisons.

Emission tax is one of the most important policy measures to reduce environmental damage due to pollution by firms. A large number of countries around the world extensively use emission taxes (Fujiwara, 2009; Antelo and Loureiro, 2009; Norregaard and Reppelinhill, 2000). Also, it is observed that most of these countries have relied substantially more on tax/subsidy policies than on other instruments such as permits (Norregaard and Reppelinhill, 2000).

Determination of optimal emission taxes on polluting firms in oligopolistic industries has received considerable attention in recent years (see Fujiwara (2009), Antelo and Loureiro (2009), Canton et al. (2008), Lahiri and Ono (2007), Yin (2003), Damania (2000), Carlsson (2000), Ulph (1996), to name a few). However, most of the existing studies consider that firms are pure profit maximizing agents and/or internal incentive structure(s) of firms are exogenously determined. The issue of strategic managerial delegation in the context of emission taxes has not received much attention in the literature so far. Moreover, the existing literature does not address the issue of how the interactions among the degree of product differentiation, strategic managerial delegation and mode of product market

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1See Requate (2005) for an excellent survey of related literature.
competition affect emission taxes. This paper attempts to fill these gaps.

Oligopolistic polluting industries with differentiated products are widely observed in real life. In addition, strategic managerial delegation is a very common phenomenon in firms that faces oligopolistic market structure. Starting with Vickers (1985), Fershtman and Judd (1987), and Sklivas (1987), the literature on strategic managerial delegation has been enriched by many studies, which examines implications of strategic delegation to various issues. However, this stream of literature neglects the issue of environmental policies and investment decision of firms to abate pollution.

We note that Barcena-Ruiz and Garzon (2002) examines the impact of strategic managerial delegation on emission taxes considering homogeneous product Cournot duopoly. It argues that a switch from pure profit maximization to strategic managerial delegation leads to higher emission taxes and environmental damage. This paper differs from Barcena-Ruiz and Garzon (2002) in two important aspects. First, this paper allows for product differentiation and examines the role of product differentiation explicitly. Second, the framework of this paper allows to examine how the interaction between strategic managerial delegation and mode of product market competition affects the optimal emission tax rate. Therefore, the analysis of this paper encompasses the analysis of Barcena-Ruiz and Garzon (2002) as a special case. In an another paper that is closely related to the present analysis, Fujiwara (2009) argues that the (positive) optimal emission tax rate is increasing in degree of product differentiation in short-run. However, Fujiwara (2009) sidesteps the issue of pollution of abatement by firms. In addition, it does not recognise the role of strategic managerial delegation and mode of product market competition.

In contrast to Fujiwara (2009), we find that (positive) optimal emission tax rate can be lower for more differentiated products, irrespective of mode of product market competition and the managerial incentive scheme offered by firms. The underlying intuition is as follows.

\footnote{For example, sequential entry (Church and Ware, 1996), mixed oligopoly (White, 2001), mergers (Gonzalez-Maestre and Lopez-Cunat, 2001; Ziss, 2001), wage bargaining (Szymanski, 1994), trade policy (Das, 1997), investment in R&D (Lambertini and Primavera, 2001; Zhang and Zhang, 1997; Kopel and Riegler, 2006; Krakel, 2004), to name a few.}
Higher degree of product differentiation leads to increase in production and, hence, higher taxes are required to control resulting increase in pollution. However, higher emission tax rate induces firms to invest more to abate pollution. The increased abatement cost, due to higher emission tax rate, adversely affects social welfare. Therefore, it is not always optimal for the government to impose higher emission tax rate in more differentiated products’ industry.

Considering a linear incentive scheme based on profits and sales, as in Fershtman and Judd (1987), this paper shows that, under quantity (price) competition, optimal emission tax rate, outputs, environmental damage and social welfare are higher (lower) in case of strategic managerial delegation than that in case of no delegation. It demonstrates that effects of strategic managerial delegation on emission tax rate, environmental damage and social welfare crucially depend on the mode of product market competition. It also implies that the results of Barcena-Ruiz and Garzon (2002) go through in case of differentiated product quantity competition, but does not hold in case of price competition.

In case of strategic managerial delegation, emission tax rate affects the output directly as well as through its impact on managerial incentive scheme. The direct effect is always negative, irrespective of the mode of product market competition and managerial delegation. On the other hand, increase in emission tax rate makes the managerial incentive scheme more profit oriented under quantity competition, but it reduces the penalty for sales maximization under price competition. As a result, under quantity (price) competition, the possibility of positive emission tax rate is more (less) in case of strategic managerial delegation than that in case of no delegation. This paper also shows that the scope for optimal emission tax rate to be lower for more differentiated products is higher in case of strategic managerial delegation than that in case of no delegation, since in case of strategic managerial delegation increase in output and corresponding increase in pollution due to increase in product differentiation is less than that in case of no delegation. These are additional insights.

The rest of the paper proceeds as follows. The next section describes the basic model. It also presents the analysis corresponding to different scenarios and illustrates the results.
Section 3 concludes.

2 The model

Let us consider an economy with an oligopolistic sector, consisting of two firms - firm 1 and firm 2, that produce a differentiated good and a competitive numeraire sector. Production technologies of firm 1 and firm 2 are identical and the associated marginal cost of production is $c$. Firm 1 and firm 2 produce $q_1$ and $q_2$ units of outputs, respectively, and are engaged either in Bertrand type price competition or in Cournot type quantity competition in the product market. The mode of competition in the product market is exogenously determined and is common knowledge.

Production of each unit of output generates one unit of pollutant. However, firms can reduce the level of pollution by adopting pollution abatement technique(s). Following Ulph (1996), we consider that firm $i$ can reduce the emission level to $(q_i - a_i)$ ($\leq q_i$) by incurring the abatement cost $C_i = \frac{a_i^2}{2}$, $i = 1, 2$. Clearly, other than price (or quantity), each firm has an additional choice variable $a_i$ ($i = 1, 2$), i.e., the level of pollution abatement.

In line with the existing literature, we consider that environmental damage ($ED$) increases at an increasing rate due to increase in pollution generated by the industry: $ED = \frac{d}{2}(q_1 - a_1 + q_2 - a_2)^2$, where $d$ ($> 0$) is the increment in marginal environmental damage due to pollution, i.e., the strength of environmental damage due to pollution. It is evident that environmental damage due to pollution reduces social welfare.

The government’s aim is to maximize social welfare ($SW$) and we consider that emission tax is the only policy instrument available to the government to correct for market distortions. Since firms do not take into account the social cost of pollution, government imposes tax $t$ per unit of pollution emission.\(^3\)

\(^3\)For simplicity, unlike Kennedy (1994), we do not distinguish between local and transboundary pollutants.

\(^4\)Antelo and Loureiro (2009), Long and Soubeyran (2005), Barcena-Ruiz and Garzon (2002) and Ulph (1996), to name a few, also consider similar environmental damage function.

\(^5\)We note here that the government can also impose environmental standards on firms (see, for example,
On the demand side of the market, we consider that the (consumption) utility function of the representative consumer is $U = q_1 + q_2 - \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2) + m$, where $m$ is the quantity of the numeraire good, $c + t < 1$.\(^6\) The degree of product differentiation is measured by the parameter $\gamma$ ($0 < \gamma < 1$), lower value of $\gamma$ denotes higher degree of product differentiation, i.e., lower degree of substitutability between products. This specification of $U(.)$ generates the following linear demand structure.\(^7\)

$$q_i = \frac{1}{1 - \gamma^2} [1 - \gamma - p_i + \gamma p_j], \quad i, j = 1, 2; i \neq j$$

Inverting (1), we get the following system of linear inverse demand functions.

$$p_i = 1 - q_i - \gamma q_j, \quad i, j = 1, 2; i \neq j$$

Clearly, social welfare, total tax collection and profit expressions are, respectively, $SW = U - c(q_1 + q_2 - \frac{a_1^2}{2} - \frac{a_2^2}{2} - ED), T = t(q_1 - a_1 + q_2 - a_2)$ and $\pi_i = (p_i - c)q_i - t(q_i - a_i) - \frac{a_i^2}{2}$.

2.1 Emission tax in case of no delegation

We begin with the scenario in which owners of firms do not hire managers. In other words, given the emission tax ($t$), owner of each firm simultaneously and independently decide the profit maximizing quantity (or price) and the level of abatement. The stages of the game involved are as follows.

Stage 1: The government decides the tax (subsidy) rate $t$ to maximize social welfare.

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\(^6\)This specification of the representative consumer’s utility function is similar to that of Singh and Vives (1984).

\(^7\)For simplicity, we assume that, unlike as in Zanchettin (2006), both firms sell positive outputs even if prices are set at marginal cost.
Stage 2: Owner of each firm simultaneously and independently decide the quantity (or price) and the level of abatement in order to maximize profit.

**Quantity competition:** We first consider that firms are engaged in Cournot type quantity competition in the product market. We solve the corresponding game by backward induction method and summarize the equilibrium outcomes in Lemma 1.

**Lemma 1:** Under quantity competition, if owners of firms do not hire managers, the equilibrium rate of emission tax, total tax collection, environmental damage, social welfare, pollution abatement levels, outputs, prices and profits are, respectively,

\[
\begin{align*}
    t_{NDQ} &= \frac{(1-c)(-1+2d(3+\gamma))}{1+\gamma+(2+\gamma)^2+2d(3+\gamma)^2}, \\
    T_{NDQ} &= \frac{2(1-c)^2(4+\gamma)(-1+2d(3+\gamma))}{(1+\gamma+(2+\gamma)^2+2d(3+\gamma)^2)^2}, \\
    ED_{NDQ} &= \frac{2(1-c)^2d(4+\gamma)^2}{(1+\gamma+(2+\gamma)^2+2d(3+\gamma)^2)^2}, \\
    SW_{NDQ} &= \frac{(1-c)^2(1+2d)(4+\gamma)}{1+\gamma+(2+\gamma)^2+2d(3+\gamma)^2}, \\
    a_{NDQ} &= a_{1,NDQ} = a_{2,NDQ} = t_{NDQ}, \\
    q_{NDQ} &= q_{1,NDQ} = q_{2,NDQ}, \\
    p_{NDQ} &= p_{1,NDQ} = p_{2,NDQ} = \frac{2+\gamma+4d(3+\gamma)+c(1+2d)(1+\gamma)(3+\gamma)}{1+\gamma+(2+\gamma)^2+2d(3+\gamma)^2}, \text{ and} \\
    \pi_{NDQ} &= \pi_{1,NDQ} = \pi_{2,NDQ} = \frac{(1-c)^2\left(11+4\gamma+2(2+\gamma)^2+12d^2(3+\gamma)^2+4d(3+\gamma)(5+2\gamma)\right)}{2\left(1+\gamma+(2+\gamma)^2+2d(3+\gamma)^2\right)^2};
\end{align*}
\]

where the subscript NDQ denotes no delegation, i.e., pure profit maximization, under quantity competition.

Note that emission tax rate affects social welfare through three channels: (a) higher tax rate leads to lower outputs, and lower outputs leads to lower social welfare; (b) higher tax rate leads to higher level of abatements and higher total abatement cost, which adversely affects social welfare; and (c) higher tax rate leads to lower emission and thus lower
environmental damage, which enhances social welfare. Therefore, whether it is optimum to impose positive tax on emission or not that depends on relative magnitudes of these effects.

It is straightforward to observe that the equilibrium emission tax rate \( t_{NDQ} \) is positive (negative), if \( d > ( <) \frac{1}{6+2\gamma} = d_{NDQ} \), say. That is, if the increment in marginal environmental damage due to pollution is more than a critical level, it is optimum to impose positive tax on emission. In other words, if the government’s concern about environmental damage is beyond a certain level, positive emission taxes are optimal. This result is in line with the existing literature on environmental tax in case of oligopolistic market structure.

Now, note that higher degree of product differentiation (i.e., lower value of \( \gamma \)) leads to higher production in each firm \( \left( \frac{\partial q_i}{\partial \gamma} < 0 \right) \). It implies that, a decrease in \( \gamma \) would lead to (a) increase in output and (b) increase in environmental damage due to higher emission. The first effect enhances social welfare, whereas the second effect adversely affects social welfare. Upon inspection we find that, \( t_{NDQ} \) is increasing (decreasing) in \( \gamma \), if \( d < (>) \frac{5+2\gamma}{2(3+\gamma)} = \bar{d}_{NDQ} \); \( d_{NDQ} < \bar{d}_{NDQ} \). It implies that the (positive) optimal emission tax rate need not necessarily be higher for more differentiated products. If the rate of marginal environmental damage due to pollution is relatively less, \( d_{NDQ} < d < \bar{d}_{NDQ} \), the optimal emission tax becomes lower as products are more differentiated. The intuition behind this result is as follows. As the degree of product differentiation increases, production increases and hence higher taxes are required to control resulting increase in pollution. However, higher tax rate leads to higher level of abatement and hence the total cost of abatement increases, which adversely affects social welfare. Note that, in equilibrium, the level of abatement in each firm is equal to the emission tax rate, i.e., the firms abate pollution up to the point where marginal abatement cost equals the tax. Therefore, unless the strength of environmental damage \( (d) \) is sufficiently high, it is optimal to have lower emission tax for more differentiated products. This result is in contrast to the result of Fujiwara (2009), where the role of pollution abatement technology is not recognised.\(^8\) It indicates that

\(^8\)If we assume away pollution abatement by firms, the optimal tax rate under Cournot competition, in no delegation case, is \( t = \frac{-1+c+2d-2cd}{1+2d+\gamma} \), which is positive for \( d > \frac{1}{2} \). Moreover, the optimal (positive) tax
pollution abatement decisions of firms and associated costs have significant bearing on impact of product differentiation on optimal emission tax rate.

**Proposition 1:** (a) Under quantity competition, in case of no delegation the optimal emission tax rate is positive, if the increment in marginal environmental damage due to pollution is greater than a critical level, \( d > d_{NDQ} = \frac{1}{6 + 2\gamma} \).

(b) In this case, the optimal emission tax rate is not necessarily increasing in degree of product differentiation. If \( d_{NDQ} < d < \bar{d}_{NDQ} = \frac{5 + 2\gamma}{2(3 + \gamma)^2} \), the optimal emission tax becomes lower as products are more differentiated. Alternatively, if \( d > \bar{d}_{NDQ} \), in equilibrium, higher degree of product differentiation leads to higher emission tax rate.

*Price competition:* We now consider Bertrand type price competition in the product market in stage 2 of the game. As before, we solve the game by backward induction method and summarize the equilibrium outcomes in Lemma 2.

**Lemma 2:** In case of price competition, if owners of firms do not hire managers, the equilibrium rate of emission tax, total tax collection, environmental damage, social welfare, pollution abatement levels, outputs, prices and profits are, respectively, rate is always decreasing in \( \gamma \).
As in case of quantity competition, under no delegation, optimal emission tax rate in case of price competition is positive (negative), if the rate of marginal environmental damage due to pollution is more than a critical level. However, since the intensity of competition in case of price competition is different from that in case of quantity competition, the critical levels of rate of marginal environmental damage beyond which the optimal emission tax rate is positive are likely to be different under alternative modes of product market competition. Upon inspection we find that, in case of price competition without delegation, the optimal emission tax rate is positive, if \( d > \frac{1 - c^2}{2(3 + \gamma - \gamma^2)} = d_{NDP} \), say. It is easy to check that, \( d_{NDP} < d_{NDQ} \). That is, the optimal emission tax rate in case of price competition is positive for lower rate of marginal environmental damage due to pollution compared to that in case of quantity competition. In other words, in case of pure profit maximization, the scope for positive emission tax rate is higher under price competition than that under quantity competition. The intuition behind this result is a switch from quantity competition to price competition leads to higher outputs as well as
higher environmental damage through expansion in pollution. To tackle the resulting pollution expansion, optimal emission tax rate becomes positive for relatively less increment in marginal environmental damage due to pollution in case of price competition than that in case of quantity competition.

It is easy to check that \( \frac{\partial PND}{\partial \gamma} > 0 \) if (a) \( \frac{1}{2} \leq \gamma < 1 \) or (b) \( \gamma < \frac{1}{2} \) and \( d < \frac{(1+\gamma)^2(5-2\gamma(3-(3-\gamma)\gamma))}{2(1-2\gamma)(3+(1-\gamma)\gamma)^2} = \bar{d}_{NDP} \). Clearly, in case of price competition with no delegation, if \( \gamma \geq \frac{1}{2} \), the (positive) emission tax rate is lower for higher degree of product differentiation. Otherwise, if \( \gamma < \frac{1}{2} \), the (positive) emission tax rate is lower for more differentiated products provided that \( d_{NDP} < d < \bar{d}_{NDP} \) holds. Therefore, in case of price competition, the impact of product differentiation on optimal tax rate is dependent on \( d \), only if the degree of product differentiation is high (\( \gamma < \frac{1}{2} \)). We summarize these results in Proposition 2.

**Proposition 2:** In case of price competition with no delegation,

(a) the optimal emission tax rate is positive, if the increment in marginal environmental damage due to pollution is more than a certain level, \( d_{PND} = \frac{1-\gamma^2}{2(3+\gamma-\gamma^2)} \); and

(b) the optimal emission tax rate is more for lower degree of product differentiation, if (i) \( \gamma \geq \frac{1}{2} \) or (ii) \( \gamma < \frac{1}{2} \) and \( d_{NDP} < d < \bar{d}_{NDP} \), where \( \bar{d}_{NDP} = \frac{(1+\gamma)^2(5-2\gamma(3-(3-\gamma)\gamma))}{2(1-2\gamma)(3+(1-\gamma)\gamma)^2} \).

Comparing the equilibrium outcomes under alternative modes of product market competition in case of no delegation, from Lemma 1 and Lemma 2, we find that emission tax rate, environmental damage, outputs and social welfare are higher under price competition than that under quantity competition. The equilibrium profits are lower, and total tax collection (when the optimal tax is positive under both quantity and price competition) is higher under price competition than that under quantity competition. Clearly, a switch from quantity competition to price competition increases outputs and thus consumer surplus, which overcompensates for the loss due to associated increase in environmental damage and decrease in profits, resulting higher social welfare.
2.2 Emission tax in case of strategic managerial delegation

Now, we consider that owners of both firms hire managers and delegate them to take decisions concerning output, as in Fershtman and Judd (1987). Owners know that managers are risk-neutral and they will engage either in Cournot type quantity competition or in Bertrand type price competition, in the product market. Thus, given the incentive structure, which is a linear combination of profits and sales, managers will maximize \( O_i = \beta_i \pi_i + (1 - \beta_i)p_iq_i, ~i = 1, 2, \) where \( \pi_i \) is the profit of firm \( i \) and \( \beta_i \) is the incentive parameter set by firm \( i \). Therefore, the stages of the game are as following.

Stage 1: The government decides the emission tax rate to maximize social welfare.

Stage 2: Owner of each firm simultaneously and independently decides the incentive parameter so that the profit is maximized.

Stage 3: Each manager chooses the level of output (or price) and abatement level, simultaneously and independently, to maximize the incentive scheme, given the incentive parameter.

We solve this game, considering quantity competition and price competition in the product market separately, by backward induction method.

Quantity competition: In case of quantity competition in the product market, given the emission tax rate and incentive parameters, the optimum choice of outputs and abatement levels of the managers in stage 3 are, respectively, \( q_i = \frac{2-\gamma-(c+t)(2\beta_i-\gamma\beta_j)}{4-\gamma^2} \) and \( a_i = t, i, j = 1, 2, i \neq j \). It implies that increase in the emission tax rate leads to less aggressive behaviour of managers. On the other hand, when \( 0 < c + t < 1 \), if owner of firm \( i \) reduces \( \beta_i \), manager of firm \( i \) become more aggressive, but its rival’s output decreases. But, if there is subsidy for emission \( (t < 0) \) and the rate of subsidy is more than \( c \) \( (c + t < 0) \), decrease in \( \beta_i \) makes the manager of firm \( i \) less aggressive. Moreover, note that in case of strategic managerial delegation also the level of abatement is chosen at a point where the marginal abatement cost is equal to the emission tax rate.
Given the emission tax rate, the equilibrium incentive parameters chosen by owners in stage 2 are $\beta_1 = \beta_2 = 1 - \frac{(1-c-t)\gamma^2}{(c+t)(4+2\gamma-\gamma^2)}$. Clearly, when $0 < c + t < 1$, the equilibrium incentive parameters are less than one. That is, in equilibrium, owners decide to make their managers to be more aggressive in the product market. However, in case of emission subsidy, owners may find it optimal to make their managers less aggressive: $\beta_i > 1$, if $c + t < 0$.

Also, note that $\frac{\partial \beta_i}{\partial t} = \frac{(1+c+t-(c+t)^2)\gamma^2}{(c+t)^2(4+2\gamma-\gamma^2)^2} > 0$, $i = 1, 2$. That is, higher emission tax rate leads to higher incentive parameters and thus to less aggressive behaviour of managers in the product market.

We now summarize the equilibrium outcomes of the game in Lemma 3.

**Lemma 3:** In case of quantity competition with strategic managerial delegation, the equilibrium rate of emission tax, total tax collection, environmental damage, social welfare, incentive parameters, pollution abatement levels, outputs, prices and profits are, respectively,

- $t_{DQ} = \frac{2(1-c)(-2 + \gamma^2 + d(12 + 2(2-\gamma)\gamma))}{4(1+\gamma) + 2d(6 + (2-\gamma)\gamma)^2 + (4 + (2-\gamma)\gamma)^2}$,
- $T_{DQ} = \frac{16(1-c)^2 (4 + \gamma - \gamma^2) (-2 + \gamma^2 + d(12 + 2(2-\gamma)\gamma))}{(4(1+\gamma) + 2d(6 + (2-\gamma)\gamma)^2 + (4 + (2-\gamma)\gamma)^2)^2}$,
- $ED_{DQ} = \frac{32(1-c)^2 d(4 + \gamma - \gamma^2)^2}{(4(1+\gamma) + 2d(6 + (2-\gamma)\gamma)^2 + (4 + (2-\gamma)\gamma)^2)^2}$,
- $SW_{DQ} = \frac{4(1-c)^2 (1 + 2d)(4 + \gamma - \gamma^2)}{4(1+\gamma) + 2d(6 + (2-\gamma)\gamma)^2 + (4 + (2-\gamma)\gamma)^2}$,
- $\beta_{DQ} = \beta_{1,DQ} = \beta_{2,DQ} = 1 - \frac{(1-c-t_{DQ})\gamma^2}{(c+t_{DQ})(4 + 2\gamma - \gamma^2)}$,
- $a_{DQ} = a_{1,DQ} = a_{2,DQ} = t_{DQ}$,
- $q_{DQ} = q_{1,DQ} = q_{2,DQ} = \frac{2(1-c)(1 + 2d)(6 + (2-\gamma)\gamma)}{4(1+\gamma) + 2d(6 + (2-\gamma)\gamma)^2 + (4 + (2-\gamma)\gamma)^2}$,
- $p_{DQ} = p_{1,DQ} = p_{2,DQ} = \frac{2(1+c+t_{DQ}) + 2(c+t_{DQ})\gamma - \gamma^2}{4 + (2-\gamma)\gamma}$, and
- $\pi_{DQ} = \pi_{1,DQ} = \pi_{2,DQ} = \frac{(2 - \gamma^2)\{4 + 4c^2 - 8c(1-t_{DQ}) - 8t_{DQ}\} + t_{DQ}^2[24 + \gamma (16 - \gamma (8 + (4-\gamma)\gamma))] - 2t_{DQ}}{2(4 + (2-\gamma)\gamma)^2}$. 

14
where the subscript \( DQ \) denotes strategic managerial delegation under quantity competition.

It is straightforward to check that the optimum emission tax rate \( (t_{DQ}) \) is positive (negative), if \( d > (<) \frac{2-\gamma^2}{2(6+2\gamma-\gamma^2)} = d_{DQ} \), say. That is, it is optimal to impose positive emission tax rate in case of strategic managerial delegation under quantity competition, if the increment in marginal environmental damage due to pollution is more than a critical level. Also, note that \( d_{DQ} < d_{NDQ} \). That is, in case of strategic managerial delegation under quantity competition the possibility of optimum emission tax rate to be positive is more than that in case of no managerial delegation.

Also, the equilibrium incentive parameter is greater than one, if \( d < d_{DQ} \) and \( 0 < c < \frac{4-2\gamma^2-4d(6+2\gamma-\gamma^2)}{(1+2d)(4+2\gamma-\gamma^2)(6+2\gamma-\gamma^2)} \). That is, if there is subsidy on emission and the marginal cost of production is less than a critical level, it is optimum for owners to make the managers less aggressive in the product market. However, such possibility does not arise, if \( d > d_{DQ} \).

Now, note that \( \frac{\partial t_{DQ}}{\partial \gamma} > 0 \), if \( d < \frac{20+\gamma(2+\gamma)(6-(2-\gamma)^2\gamma)}{2(1-\gamma)(6+2\gamma-\gamma^2)^2} = \bar{d}_{DQ} \). That is, the optimal emission tax rate is lower (higher) for more (less) differentiated products, if the increment in marginal environmental damage is less than certain level \( (\bar{d}_{DQ}) \), in case of strategic managerial delegation under quantity competition. The underlying intuition is similar to that in case of no delegation. However, note that \( \bar{d}_{DQ} > \bar{d}_{NDQ} \). It implies that under quantity competition the scope for the optimal emission tax rate to be lower (higher) for more (less) differentiated products is greater in case of strategic managerial delegation than that in case of no delegation. Because, in case of strategic managerial delegation, increase in output and associated increase in pollution level due to increase in product differentiation is less than that in case of no delegation \( (0 > \frac{\partial q_{DQ}}{\partial \gamma} > \frac{\partial q_{NDQ}}{\partial \gamma}) \). It also implies that the rate of change in emission tax rate due to change in product differentiation is higher in case of strategic managerial delegation than that in case of no delegation: \( \frac{\partial t_{DQ}}{\partial \gamma} > \frac{\partial t_{NDQ}}{\partial \gamma} \).

**Proposition 3:** (i) In case of strategic managerial delegation under quantity competition,

(a) the optimal emission tax rate is positive, if the increment in marginal environmental damage due to pollution is higher than the critical level \( d_{DQ} \) \( (= \frac{2-\gamma^2}{2(6+2\gamma-\gamma^2)}) \); and
(b) the optimal emission tax rate is more for lower degree of product differentiation, if $d < \bar{d}_{DQ}$, where

$$\bar{d}_{DQ} = \frac{20+\gamma(2+\gamma)(6-(2-\gamma)^2)}{2(1-\gamma)(6+2\gamma-\gamma^2)^2}.$$ 

(ii) Under quantity competition, (a) the scope for the optimal emission tax rate to be lower (higher) for more (less) differentiated products and (b) the possibility of optimum emission tax rate to be positive are higher in case of strategic managerial delegation than that in case of no delegation.

Now, we turn to compare the equilibrium outcomes, under quantity competition, in case of strategic managerial delegation with that in case of no delegation.\(^9\) It is easy to check that if emission tax is positive, the equilibrium incentive parameter in case of strategic managerial delegation, under quantity competition, is less than one, $\beta_{DQ} < 1$, which induces the managers to be more aggressive in the product market and produce more outputs than that in case of no delegation. This in turn invites higher emission tax rate in case of strategic managerial delegation than that in case of no delegation: $t_{DQ} > t_{NDQ}$. However, it turns out that the positive effect of incentive scheme outweighs the negative effect of increased emission tax rate on outputs and thus the equilibrium outputs are higher in case of strategic managerial delegation than that in case of no delegation ($q_{DQ} > q_{NDQ}$). Therefore, in equilibrium, under quantity competition consumer surplus and total tax collection are higher, but profits are lower, in case of strategic managerial delegation than that in case of no delegation. Also, it is easy to check that $q_{DQ} - q_{NDQ} > t_{DQ} - t_{NDQ}$, which indicates that the marginal environmental damage due to pollution under quantity competition is higher in case of strategic managerial delegation compared to that in case of no delegation. Thus, under quantity competition the equilibrium environmental damage in case of strategic managerial delegation is more than that in case of no delegation. Upon inspection we find that, greater consumer surplus and total tax collection together over

\(^9\)Note that, if $t < 0$, we would have $a_1 = a_2 = 0$, $\pi_i = (p_i - c)q_i - t q_i$, $ED = \frac{1}{2}d(q_1 + q_2)^2$ and $SW = U - c(q_1 + q_2) - ED$, respectively. Clearly, the optimal profits, environmental damage and social welfare in case of subsidy would be different from that in case of (positive) tax. However, the optimal outputs, prices, and incentive parameters will remain the same.
compensate for higher environmental damage and lower profits. Therefore, social welfare under quantity competition is higher in case of strategic delegation than that in case of no delegation.

**Proposition 4:** In case of strategic managerial delegation, under quantity competition, the equilibrium profits of firms are lower, but emission tax rate, outputs of firms, total tax collection, environmental damage and social welfare are higher than that in case of no delegation.

**Price competition:** We now turn to analyse the case of strategic managerial delegation under price competition. In this case, in stage 3 of the game the manager of firm $i$ sets the price $p_i = \frac{(2-\gamma-\gamma^2) + (c+t)(2\beta_1+\gamma\beta_2)}{4-\gamma^2}$ and pollution abatement level $a_i = t$, $i,j = 1,2$, $i \neq j$. It implies that increase in emission tax rate leads to increase in prices as well as levels of abatement set by managers. It implies that increase in emission tax rate induces the managers to be less aggressive in the product market. Moreover, when $c+t > 0$ ($c+t < 0$), if owner of firm $i$ reduces $\beta_i$, manager of firm $i$ becomes more (less) aggressive, as in case of quantity competition.

However, given the emission tax rate, the equilibrium incentive parameters chosen by owners in stage 2 are $\beta_1 = \beta_2 = 1 + \frac{(1-c-t)(1-\gamma)\gamma^2}{(c+t)(4-2\gamma-\gamma^2)}$. That is, when $1 > c + t > 0$, under price competition owners choose the incentive parameter to be greater than one, $\beta_i > 1$, which restricts the managers’ aggressiveness in the product market. Alternatively, if there is subsidy on emission and $c+t < 0$, the optimal incentive parameter is less than one, which induces the managers to be more aggressive. Also, note that $\frac{\partial \beta}{\partial t} = - \left( \frac{(1-\gamma)\gamma^2}{(c+t)^2(4-2\gamma-\gamma^2)} \right) < 0$. That is, the optimal incentive parameter is decreasing in emission tax rate. Clearly, these results are opposite to that under quantity competition. Lemma 4 summarizes the equilibrium outcomes of the game under price competition.

**Lemma 4:** In case of strategic managerial delegation under price competition, the equilibrium rate of emission tax, total tax collection, environmental damage, social welfare, incentive parameters, pollution abatement levels, outputs, prices and profits are, respec-
tively,

\[ t_{DP} = \frac{2 (1 - c) (2 - \gamma^2) (-1 + \gamma^2 + d (6 + \gamma (2 - \gamma (4 + \gamma))))}{(1 + \gamma) (20 - 24 \gamma^2 + 6 \gamma^4 + \gamma^5) + 2 d (6 + \gamma (2 - \gamma (4 + \gamma))))^2}, \]

\[ T_{DP} = \frac{4 (1 - c)^2 (2 - \gamma^2)^2 (8 + \gamma (2 - \gamma (6 + \gamma))) (-1 + \gamma^2 + d (6 + \gamma (2 - \gamma (4 + \gamma))))}{(1 + \gamma) (20 - 24 \gamma^2 + 6 \gamma^4 + \gamma^5) + 2 d (6 + \gamma (2 - \gamma (4 + \gamma))))^2}, \]

\[ E_{DP} = \frac{2 (1 - c)^2 d (2 - \gamma^2)^2 (8 + \gamma (2 - \gamma (6 + \gamma))))}{(1 + \gamma) (20 - 24 \gamma^2 + 6 \gamma^4 + \gamma^5) + 2 d (6 + \gamma (2 - \gamma (4 + \gamma))))^2}, \]

\[ SW_{DP} = \frac{(1 - c)^2 (1 + 2 d) (2 - \gamma^2) (8 + \gamma (2 - \gamma (6 + \gamma))))}{(1 + \gamma) (20 - 24 \gamma^2 + 6 \gamma^4 + \gamma^5) + 2 d (6 + \gamma (2 - \gamma (4 + \gamma))))^2}, \]

\[ \beta_{DP} = \beta_{1,DP} = \beta_{2,DP} = \frac{(1 - \gamma) \gamma^2 + c (2 - \gamma) (2 - \gamma^2) - t_{DP} (2 - \gamma) (2 - \gamma^2)}{(c + t_{DP}) (4 - \gamma (2 + \gamma))}, \]

\[ a_{DP} = a_{1,DP} = a_{2,DP} = t_{DP}, \]

\[ q_{DP} = q_{1,DP} = q_{2,DP} = \frac{(1 - c) (1 + 2 d) (2 - \gamma^2) (6 + \gamma (2 - \gamma (4 + \gamma))))}{(1 + \gamma) (20 - 24 \gamma^2 + 6 \gamma^4 + \gamma^5) + 2 d (6 + \gamma (2 - \gamma (4 + \gamma))))^2}, \]

\[ p_{DP} = p_{1,DP} = p_{2,DP} = \frac{2 (1 + c + t_{DP}) - 2 \gamma - (c + t_{DP}) \gamma}{4 - \gamma (2 + \gamma)}, \] and

\[ \pi_{DP} = \pi_{1,DP} = \pi_{2,DP} = \frac{1}{2 (1 + \gamma) (4 - \gamma (2 + \gamma))} [(1 - \gamma) (2 - \gamma^2) (4 + 4 c^2 - 8 c (1 - t_{DP}) - 8 t_{DP}) + t_{DP}^2 (24 + \gamma (-8 + \gamma (-24 + \gamma (1 + \gamma) (4 + \gamma)))]. \]

where the subscript \( DP \) denotes strategic managerial delegation under price competition.

Note that, \( t_{DP} > 0 \Leftrightarrow d > \frac{1 - \gamma^2}{6 + \gamma (2 - 4 \gamma - \gamma^2)} = d_{DP}. \) That is, the optimal emission tax rate is positive in case of strategic managerial delegation under price competition, if the rate of marginal environmental damage due to pollution is greater than \( d_{DP}. \) It is easy to check that \( d_{DP} > d_{NDP}. \) It implies that, unlike as under quantity competition, the possibility of optimal emission tax rate under price competition to be positive is less in case of strategic managerial delegation than that in case of no delegation.

Also, note that the equilibrium incentive parameter under price competition is not necessarily always greater than one. If \( d < d_{DP} \) and \( 0 < c < \frac{1 - \gamma^2 - d (6 + \gamma (2 - 4 \gamma - \gamma^2))}{(1 + 2 d) (1 + \gamma) (4 - 2 \gamma - \gamma^2) (6 + \gamma (2 - 4 \gamma - \gamma^2))}, \) \( \beta_{DP} < 1. \)

Finally, we observe that \( \frac{\partial a_{DP}}{\partial \gamma} > 0, \) if (a) \( 0.636203 \leq \gamma \leq 1 \) or (b) \( \gamma < 0.636203 \) and \( d < \frac{(1 + \gamma)^2 (40 - 12 \gamma - 3 \gamma^2 + 2 \gamma^3)}{(6 + 2 \gamma - 4 \gamma^2 - \gamma^3) (4 - 4 \gamma - 2 \gamma^2 + \gamma^3)} \) = \( \tilde{d}_{DP}. \) Therefore, in case of price competition with strategic managerial delegation, if \( \gamma \geq 0.636203 \) the (positive) emission tax rate is lower for higher degree of product differentiation. Otherwise, if \( \gamma < 0.636203, \)
the (positive) emission tax rate is lower for more differentiated products provided that $d_{DP} < d < \bar{d}_{DP}$ holds. It is easy to check that $\bar{d}_{DP} < \bar{d}_{NDP}$ ($\bar{d}_{DP} > \bar{d}_{NDP}$), if $\gamma > 0.636203$ ($\gamma < 0.636203$). Comparing it with Proposition 2, it is evident that under price competition the possibility of the (positive) emission tax rate to be lower for higher degree of product differentiation is less (more) in case of strategic managerial delegation than that in case of no delegation, if the degree of product differentiation ($\gamma$) is greater (less) than $\frac{1}{2}$.

**Proposition 5:** (i) In case of price competition with strategic managerial delegation, (a) the optimal emission tax rate is positive, if the increment in marginal environmental damage due to pollution is greater than $d_{DP}$ ($= \frac{1-\gamma^2}{6+\gamma(2-4\gamma)} > d_{NDP}$); and (b) the optimal emission tax rate is more for lower degree of product differentiation, if (1) $\gamma \geq 0.636203$ or (2) $\gamma < 0.636203$ and $d < \bar{d}_{DP}$, where $\bar{d}_{DP} = \frac{(1+\gamma)^2[40-(2-\gamma)\gamma(28+\gamma)(10-12\gamma+3\gamma^2+2\gamma^3)]}{(6+2\gamma-4\gamma^2-\gamma^3)^2(4-4\gamma-4\gamma^2+\gamma^3)}$.

(ii) In case of price competition with strategic managerial delegation the possibility of optimal emission tax rate to be positive is less than that in case of price competition without delegation. Moreover, under price competition, the possibility of the optimal emission tax rate to be lower for higher degree of product differentiation is less in case of strategic managerial delegation than that in case of no delegation, unless the degree of product differentiation is less than $\frac{1}{2}$.

From Proposition 3 and 5, it is evident that strategic managerial delegation affects the critical amount of increment in marginal environmental damage due to pollution, for optimal emission tax rate to be positive, in opposite directions under alternative modes of product market competition. Impacts of strategic managerial delegation on the scope for the optimal emission tax rate to be lower for higher degree of product differentiation under alternative modes of product market competition are also quite different.

Now, comparing Lemma 2 and Lemma 4, we find that the optimal emission tax rate, under price competition, in case of strategic managerial delegation is less than that in case
of no delegation: $t_{DP} < t_{NDP}$. This is in sharp contrast to that under quantity competition. The reason is, under price competition with strategic managerial delegation, when the optimal emission tax rate is positive, the equilibrium incentive parameter is greater than one ($\beta_{DP} > 1$), which induces the managers to be less aggressive in the product market than that in case of no delegation and thus invites lower emission tax rate. Note that, ceteris paribus, outputs are decreasing in emission tax rate. However, under price competition, the optimal incentive scheme reduces the outputs of firms by more than the amount of increase in outputs due to lower taxes in case of strategic managerial delegation from that in case of no delegation. Therefore, in equilibrium, outputs are lower in case of price competition with strategic managerial delegation than that in case of price competition without delegation. This in turn leads to lower consumer surplus and lower total tax collection and higher profits in case of strategic managerial delegation compared to that in case of no delegation, under price competition. Now, it is easy to check that $q_{DP} - q_{NDP} < t_{DP} - t_{NDP}$. It implies that the marginal environmental damage due to pollution, under price competition, is lower in case of strategic managerial delegation than that in case of no delegation. Therefore, under price competition, in equilibrium, environmental damage in case of strategic managerial delegation is less than that in case of no delegation. Finally, we find that social welfare in case of price competition with strategic managerial delegation is less than that in case of price competition with no delegation, since the combined loss due to lower consumer surplus and lower total tax collection dominates the total gain due to higher profits and lower environmental damage due to pollution.

**Proposition 6:** In case of strategic managerial delegation, under price competition, the equilibrium profits of firms are higher, but emission tax rate, outputs of firms, total tax collection, environmental damage and social welfare are lower than that in case of no delegation.

The above proposition is in sharp contrast to Proposition 4. That is, the impact of strategic managerial delegation on optimal emission tax rate, environmental damage
and social welfare are in opposite directions under alternative modes of product market competition, quantity and price. Nonetheless, comparing Lemma 3 and Lemma 4 we find that the equilibrium emission tax rate, environmental damage, and social welfare are higher under price competition than that under quantity competition even in case of strategic managerial delegation.

3 Conclusion

This paper analyses the impacts of product differentiation, strategic managerial delegation and mode of product market competition, as well as of their interactions, on optimal emission tax rate, environmental damage and social welfare. The analysis of this paper helps to understand the underlying mechanism to design emission tax scheme in a broader framework. This paper clearly demonstrates that mode of product market competition, incentive schemes within firms, degree of product differentiation and strength of environmental damage due to pollution as well as pollution abatement cost play vital roles in determining the appropriate environmental policy scheme.

It shows that, in contrast to Fujiwara (2009), (positive) optimal emission tax rate can be lower for more differentiated products under plausible parametric configurations. It also shows that under quantity (price) competition, the possibility of positive emission tax rate is more (less) in case of strategic managerial delegation than that in case of no delegation; and the scope for optimal emission tax rate to be lower for more differentiated products is higher in case of strategic managerial delegation than that in case of no delegation. Moreover, this paper shows that under quantity (price) competition, optimal emission tax rate, outputs, environmental damage and social welfare are higher (lower) in case of strategic managerial delegation than that in case of no delegation.

Note that this paper considers a linear combination of profit and sales revenue as the managerial incentive scheme. This particular incentive scheme is widely used in the literature. Nonetheless, in reality the managerial incentive scheme can be of different forms as well, such as a linear combination of firm’s own profit and its rival’s profit (Miller and
Pazgal, 2001), weighted sum of profits and market share (Jansen et al., 2007), etc. It is straightforward, but tedious, to check that the qualitative results of this paper remain valid under alternative incentive schemes as well. To illustrate it further, note that, if owners design incentive schemes based on a linear combination of its own profit and its rival’s profit, price and quantity competition leads to same equilibrium outputs in case of strategic managerial delegation (Miller and Pazgal, 2001), which lie in between the equilibrium outcomes in case of no delegation under price competition and quantity competition. Clearly, given the mode of product market competition, the direction of change in equilibrium outputs due to a switch from no delegation to strategic managerial delegation is not sensitive to the type of managerial incentive scheme. It indicates that, the qualitative results of this paper seems to be robust to the form of managerial incentive scheme.

In this paper, we have considered closed economies. In case of open economies, strategic behaviour of governments in designing environmental policies to control local and global pollution ads additional dimensions. It seems to be interesting to extend the present analysis by considering open economies and strategic behaviour of governments.

References


