Incomplete Contracts, Incentives and Economic Power

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Abstract
This paper formalizes ideas from classical and radical political economy on task allocation and technology adoption under capitalism. A few previous studies have attempted this, but the framework and results in this paper are different. I model labor contracts that are incomplete owing to unforeseen/indescribable contingencies, leading to Pareto-improving renegotiation and a hold-up problem. Given path dependence, the allocation is sub-optimal, with the extent of inefficiency depending upon the degree of incompleteness. This model captures insights from the above literature on the microeconomic roots of inefficiency and power. It also provides a concrete setting where indescribable contingencies do (or don’t) matter - a much-debated issue.

Keywords:
Incomplete Contracts; Unforeseen/Indescribable Contingencies; Hold-Up; Classical and Radical Political Economy

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1 Introduction

The literature on incomplete contracts\(^1\) has given us insights into difficult concepts like ownership (e.g. Grossman and Hart 1986) and power,\(^2\) and also provided a bridge between different paradigms within economics.\(^3\) The latter accomplishment is in my opinion relatively underappreciated, and hence in this paper, I attempt to focus on it. To be more specific, I use ideas from the incomplete contracting literature to explore an issue (allocation of tasks and adoption of technology in an employment relationship) that has received considerable attention in classical and radical political economy. In this process, I also try to shed light on one dimension of incompleteness (indescribability) on which there is considerable debate.

While it is well known that Adam Smith extolled division of labor and specialization (e.g. in his famous discussion on the pin factory), it is not equally well known that he was concerned about the adverse effects on laborers of performing a narrow set of tasks, and made a case for government intervention to address this.\(^4\) Adam Ferguson, another Scottish philosopher and a contemporary of Adam Smith wrote

\(^1\)For the purposes of the paper, an incomplete contract refers to any exchange in which some aspect of a transaction is either unspecified, or if specified, not enforceable costlessly. This includes cases of hidden information or actions (e.g. moral hazard). I discuss the various sources of incompleteness below.

\(^2\)I discuss this issue in detail and provide references below.

\(^3\)When incompleteness of contracts is introduced, some results derived from different paradigms are similar. Also, a space for conversation is created between people working within different paradigms. For example, the work on “contested exchange” by Bowles and Gintis (1993), although inspired by Marxian and radical political economy, is similar in spirit to the standard principal-agent model. Also see Bowles (1985) on different theories of the firm, and the discussion in Milgrom and Roberts (1992, pp. 256-7).

\(^4\)“... His dexterity at his own particular trade, seems in this manner to be acquired at the expense of his intellectual, social, and martial virtues. But in every improved and civilized society this is the state into which the labouring poor, that is, the great body of the people must necessarily fall, unless government takes some pains to prevent it ...” Smith 1776, Book V, p. 369.
extensively on the negative effects of division of labor and work under capitalism.\(^5\) Although not well known among economists, his work influenced modern sociology and several thinkers, including Marx (Skinner (1979, p. 97); Hill 2007). Marx\(^6\) and other authors working in the Marxian tradition have argued that the organization of production under capitalism (e.g. the factory system) is concerned more with disciplining the workforce than with efficiency, e.g. in an influential contribution, Marglin (1974) argued that the putting-out and factory systems rather than being more efficient, enriched the capitalists at the expense of workers.\(^7\) Another author, Braverman (1974) has argued that a process of routinization and deskilling of work has occurred in the twentieth century and this process has made workers worse-off and benefited employers. Some authors have used these ideas to examine particular industries, e.g. see Noble (1978) on machine tools.\(^8\) Denrell (2000) provides several references to argue that these ideas percolated from Marxian theory to modern organizational theory.

While the above literature (especially the work of classical writers) is rather rich and prone to multiple interpretations, the purpose of this paper is more modest.

It intends to take two of its claims seriously: (a) Employment contracts may be

\(^5\)"Many mechanical arts require no capacity; they succeed best under a total suppression of sentiment and reason ... manufactures prosper most where the mind is least consulted, and where the workshop may ... be considered as an engine, the parts of which are men...” Ferguson (1767) cited from Hill (2007).

\(^6\)I have in mind both the idea from historical materialism that the institutional structure (“relations of production”) constrains productive potential/technology (“forces of production”) and the possibility of technical change being biased, e.g. see the discussion in Marx (1867 [1977], Chapter 15) on “Machinery and Large-Scale Industry,” especially section 3.

\(^7\)“...Rather than providing more outputs for the same inputs, these innovations in work organization were introduced so that the capitalist got himself a larger share of the pie at the expense of the worker ...” Marglin 1974.

\(^8\)Also see other articles in this particular issue of the journal *Politics and Society.*
unable to optimally resolve the conflict of interest between laborers and employers;
(b) the adoption by employers of tasks, technology, and practices in general, is
dictated by their desire to exercise power and control over the production process
and laborers. A natural question that arises in this context is that since employers
are basically interested in maximizing their profits, what necessitates them to adopt
such practices. Moreover, how do we rigorously formalize the idea that employers
exercise power or control? This paper tries to explore these questions using the
language of modern microeconomics. It asks the following questions: if we want to
take the above claims seriously, in light of modern microeconomics, how do we do
so?; What are the conditions under which these claims hold? It is my contention
that within the paradigm of incomplete contracting, these claims can be explored in
a meaningful manner. Also, some plausible conditions can be derived under which
these claims hold.

While some authors have tried to formalize the above ideas without relying
on incomplete contracting,\(^9\) the literature that is relevant for this paper, uses in-
complete contracts. For example, in a seminal paper, Bowles (1985) draws upon
the distinction made by Marx between labor (work or effort) and labor power (la-
bor time) and between the spheres of exchange (the marketplace) and production
(within the firm). Since effort is non-contractible, employers face the problem of
extracting it from workers, which they do by surveillance (which is costly) and by
threatening to fire laborers who are caught shirking. Within the context of this

\(^9\)For example, Bhadhuri (1973) formalizes Marx’s insights by exploring the adoption of technology by
a landlord-moneylender within the context of semi-feudal/less developed agriculture. The landlord leases
out land and lends to a poor sharecropper-borrower. Bhadhuri shows how the landlord-moneylender has
an incentive to adopt inferior technologies and keep the tenant indebted to him. Roemer (1983) shows
that it is optimal for profit-maximizing employers to discriminate, i.e. adopt divide-and-rule practices
towards laborers. For other related references, see Bowles (1985).
model, Bowles shows how profit maximizing employers can adopt inefficient technologies or divide-and-rule strategies. In this class of labor-discipline or efficiency wage models, the employer’s power over the laborer stems from the result that the employer can credibly threaten to fire the laborer, whereas a similar threat by the laborer (to leave the employer) is not credible. This is consistent with both popular notions of power\textsuperscript{10} and with academic literature\textsuperscript{11}.

Another relevant contribution is Denrell (2000) (which is based upon a model of non-binding labor contracts by Grout (1984)), where the employer assigns tasks to the laborer and also makes a specific non-contractible investment which increases in the discretion that is entrusted to the laborer. This gives rise to a hold-up problem, anticipating which the employer underinvests by assigning sub-optimal tasks to the laborer. Denrell’s interpretation is that the employer’s choice reflects his/her desire to increase his/her power or control by reducing his/her dependence upon the laborer. Power is equated with lack of dependence, which is consistent with the literature in sociology and radical organizational theory. Within debates in economics too, specificity and dependence have played an important role. For example, in an influential contribution, Alchian and Demsetz (1972) have argued that a firm’s power is no different from what prevails in market transactions, e.g.

\textsuperscript{10} See e.g. Francois Truffaut’s murder mystery \textit{Finally Sunday}, where one of the characters remarks: “It’s really unfair. If a boss can fire me, why can’t I fire him?”

\textsuperscript{11} For a survey of the relevant literature, see Bowles (2004, pp. 338-349), who argues that a sufficient condition for the exercise of power is that: “For B to have power over A it is sufficient that by imposing or threatening to impose sanctions on A, B is capable of affecting A’s actions in ways that advance B’s interests, while A lacks this capacity with respect to B” (p. 345). This condition is satisfied in the context of efficiency wage models. This issue has to be seen in light of Samuelson’s (1957) remark that in a Walrasian (or Arrow-Debreu) world, there is no qualitative distinction between capital and labor - each can hire and fire the other
the relationship between a grocer and its employee is no different from that between the grocer and its customer. Hart’s (1995, pp. 57-58) response that this is not the case hinges on the idea that the employer can deprive the employee of the assets that he/she needs. In this sense, the employee depends upon the employer. As this debate illustrates, both these notions of power are not entirely unrelated to each other - the employee who has made a specific investment cannot credibly threaten to leave his/her employer, whereas the employer can credibly threaten to fire him/her.

Although related to the above studies (especially to Denrell (2000)), this paper departs from them in some key respects. First, I focus on one source of incompleteness that has not been explored in the above literature. This concerns “unforeseen” or “indescribable” contingencies wherein economic agents are unable to foresee or physically describe future contingencies that are relevant to them. Apart from being the more realistic assumption in at least some contexts, I believe that the introduction of unforeseen contingencies, leads to interesting implications. Second, I draw on the strand of literature that explores the resolution of conflicts of interest in the presence of incomplete contracts. In particular, Aghion and Bolton (1992) analyze financial contracts between a venture capitalist who cares only for the monetary return on a project and an entrepreneur who cares about not only the monetary return, but also other attributes (e.g. being his/her own boss etc.). The entrepreneur’s costs and benefits are affected by future events - the contract cannot be based upon these events, but on a signal correlated with them. Overall,

12Some reasons for incompleteness that have been highlighted in the literature (Bowles (2004, p. 236); Maskin 2002) are: (i) Information is not available to all parties, so that it can be verified by a third party, (ii) Absence of a third party, (iii) It is impossible to specify all contingencies, (iv) It is too expensive to specify all contingencies, (iv) It is in the interest of party(ies) to offer incomplete contracts

13See Anderlini and Felli (1994), which argues that indescribability is linked to the algorithmic nature of contracts and the need for making formal arguments.
as I describe in detail below, the modeling framework used and results obtained in this paper differ from those in previous studies.

Before presenting the technical details of the model developed in this paper, it is worthwhile to summarize the basic setup and results. There is an employer and a laborer. The employer has a project and faces a choice over the processes that he/she can adopt. A process specifies the set of tasks that the laborer has to perform and the technology, i.e. the returns from the project for performing these tasks. Processes vary in sophistication and if an employer adopts a process, he/she needs to make a specific non-contractible investment. The more sophisticated a process, the higher is this investment. Consider an example: a firm needs to hire a worker to record its inventory. A low sophistication process could involve hiring a literate person and asking him/her to use pen and paper, whereas a high sophistication process could involve the worker using a computer based inventory management system after receiving adequate training. While the employer cares only about his/her monetary share of the returns from the project, the laborer cares both about his/her share and the process (i.e. the tasks that are assigned to him/her). The laborer may not want to do dull/mundane tasks, or may prefer discretion, or may not want too much responsibility, or may like the training/learning etc. The laborer and employer are interacting in a complex world - neither of them can correctly foresee (or describe) the true state of the world in the future, however both of them have access to an imperfect signal that they can use. Once they learn about the true nature of the world, there can be renegotiation by reassigning processes/tasks and respecifying the shares of the surplus of the employer and the laborer. Given this, there is a trade-off - on the one hand, there is scope for Pareto-improvement once the true nature of the world is known, but on the other hand, since the employer makes a specific investment, he/she will lose this if the
laborer walks away during renegotiation, and this will lead him/her to underinvest by assigning a less sophisticated/sub-optimal process (akin to the standard hold-up problem). If the reassignment of processes is costly (as is realistic to assume) and there is path dependence (the specific form of path dependence is clarified below), then the allocation is not first-best or even constrained first-best (which is defined as first best among allocations which use the imperfect signal in a world with unforeseen contingencies). The extent of the inefficiency, i.e. difference from the first-best can be linked to the degree of incompleteness.

The analysis formalizes the idea from the above-mentioned literature that the organization of production can be suboptimal given incomplete contracts and conflicts of interest in the employment relationship. Although the paper relies on the hold-up problem and the interpretation of power as dependence, there are some key differences from previous studies. These are elaborated in section 3, but it is worthwhile to summarize these here. First, the modeling framework and therefore the results are different. To reiterate, I focus on a source of incompleteness (viz., indescribability) that has not been explored earlier, and thereby obtain new results - I am able to highlight the role of path dependence and link the degree of incompleteness to the extent of inefficiency. Second, certain mechanisms that can be used to address the hold-up problem in standard models will not work here. e.g. if parties are prevented from renegotiating then there is no hold-up problem, but there is also no scope for Pareto improvements, so we do not obtain first-best (as would be the case in a standard model) or constrained first-best.

The literature on indescribable contingencies (e.g. Hart and Moore 1999; Maskin 2002) has been quite contested, with several relevance and irrelevance results. Given that this literature is fairly abstract, the analysis in the paper can be seen as providing a concrete example where indescribable contingencies do and do not matter.
The remaining part of the paper is organized into two sections. Section 2 presents the model and the analysis. Section 3 discusses the results and concludes.

2 The Model

2.1 Setup

There is a risk neutral employer who has access to a project for which he/she needs a laborer. The employer can choose from a continuum of processes, \( \mathcal{P} = [P, \overline{P}] \) where \( P \) and \( \overline{P} \) denote processes that are least and most sophisticated, respectively. A process \( P \in \mathcal{P} \) is associated with a triple: \( (y(P; \theta), t(P), i(P)) \) with \( y(P; \theta) \) denoting the returns from \( P \). \( \theta \) is a stochastic variable that represents the true nature of the world. One can interpret it as the “fit” among the process, the laborer and the employer, which can be excellent, good, bad etc. \( y \) is continuous and differentiable and is increasing, but at a decreasing rate in the sophistication of the process assigned. Also, the better the fit, the higher is the output and the marginal output (with respect to the process), i.e.,

\[
\frac{\partial y}{\partial P} > 0, \, \frac{\partial^2 y}{\partial P^2} < 0, \, \frac{\partial y}{\partial \theta} > 0, \, \frac{\partial y}{\partial \theta \partial P} > 0
\] (1)

The fit cannot be foreseen or described at the outset by the employer or the laborer. Both of them are aware of this fact, and have access to an imperfect signal \( (s) \) that they can use. Once (and only after) a process is assigned and some time elapses, the true nature of the fit will be known. The idea here is that in a complex world, both the employer and the laborer can only discover the fit by initially working together. Let \( \theta \in A \) and has a cumulative distribution function \( F_\theta \), whereas \( s \in A \) and has a cumulative distribution function \( F_s \). The difference between \( \theta \) and \( s \) can be thought of as a measure of the incompleteness of contracts since only \( s \) (and
not \( \theta \) can be incorporated into contracts. Alternatively, it can also be thought of as a measure of complexity of the real world.\(^{14}\) I conceptualize the degree of incompleteness \( D(H(\cdot)) \) in the following manner:

\[
D(H(\cdot)) = \left| \int_A H(x)F_{\theta}(x) - \int_A H(s)F_s(x) \right| \tag{2}
\]

\( H(x) \) is a function of either \( s \) or \( \theta \) (i.e. \( x = s \) or \( \theta \)) and could for example be the output, cost or marginal cost. Note that \( D \) measures the extent to which the world differs from using the signal \( s \) instead of using the true fit \( \theta \). The sequence of events is described in figure 1. At date \( \frac{1}{2} \), once the true distribution of \( \theta \) is revealed, there is a possibility for renegotiation. This issue will be discussed in detail below.

\( t(P) \) denotes the set of tasks assigned to the laborer. To focus on the main issues, I abstract away from considerations of moral hazard and assume that the laborer’s work (i.e. tasks) can be monitored at no cost. \( i(P) \) denotes the investment that the employer has to make, which could represent for example costs incurred in training the laborer, or setting up infrastructure. I assume that \( i(P) \) is continuous and differentiable. The more sophisticated the process, the higher is the investment, which is consistent with the interpretation that more sophisticated processes involve allocating more skilled tasks or more discretion (and hence more/better training or better infrastructure) to the laborer. I assume increasing marginal investment and normalize the least sophisticated process to require no investment, i.e.,

\[
\frac{\partial i(P)}{\partial P} > 0, \quad \frac{\partial^2 i(P)}{\partial P^2} > 0, \quad i(P) = 0 \tag{3}
\]

Let \( w \) denote the wage that the employer pays to the laborer. The profit of the

\(^{14}\)Aghion and Bolton (1993) use a similar idea, although they analyze the conflict of interest between a wealthy investor and an entrepreneur. The former cares only about pecuniary returns from the project, whereas the latter cares about both pecuniary and non-pecuniary returns.
employer from choosing process \( P \) is therefore:

\[
\pi(P) = y(P; \theta) - i(P) - w \tag{4}
\]

To abstract away from considerations of risk-sharing and focus on the main issues, I assume that the laborer is also risk neutral. I also assume that the laborer is wealth-constrained so that he/she cannot buy the project from the employer. The reservation utility of the laborer is given by \( u \). The laborer cares about his/her wage and the process that is assigned to him/her. If the employer pays a wage \( w \) and chooses process \( P \), the laborer’s utility is given by:

\[
u = w + g(P) \tag{5}\]

g represents the satisfaction that the laborer gets from process \( P \). While it is not essential for the results, for ease of exposition, I assume that the laborer likes discretion and therefore his/her utility increases in the sophistication of the process assigned, although at a decreasing rate:

\[
\frac{\partial u}{\partial P} = \frac{\partial g(P)}{\partial P} > 0, \quad \frac{\partial^2 u}{\partial P^2} = \frac{\partial^2 g(P)}{\partial P^2} < 0 \tag{6}\]

At date \( \frac{1}{2} \), when the true nature of \( \theta \) is revealed, parties realize the extent to which the signal they used departs from the true value, or in other words, the degree of incompleteness of the contract. This gives rise to the possibility of renegotiation over wages and the process assigned. For simplicity, and as is standard in the literature, we assume that the investment is specific, i.e. if the laborer leaves after the investment is made, the employer loses it. This assumption is not crucial - as long as some degree of specificity exists, we can show that the results go through. If the laborer stays, but if the process is reassigned, say from \( P_1 \) to \( P_2 \), then there is some cost of/loss from doing so. If \( P_2 \) is more sophisticated than \( P_1 \) then the additional investment needed for the reassignment is \((i(P_2) - i(P_1) + c(P_1, P_2))\). If \( P_2 \)
is less sophisticated than $P_1$ then $c(P_1, P_2)$ represents the part of the investment that cannot be recovered. I assume that the cost function is well behaved, continuous and differentiable. As is natural in this context, I also assume that if there is no reassignment of processes, then no cost is incurred, otherwise, there is a positive cost. This is summarized below:

(C1) $C$ is continuous, differentiable and satisfies the following property:

$$c(P_1, P_2) = 0, \quad P_1 = P_2; \quad P_1, P_2 \in \mathcal{P}$$

$$> 0 \quad \text{otherwise}$$

(C1) implies that the cost function cannot depend solely on either the initial ($P_1$) or the final ($P_2$) process.\(^{15}\) Hence, the first derivatives of $c(., .)$ cannot vanish everywhere. As we see below, this implication will be useful. I will also assume that the first derivatives of the cost function are continuous and differentiable:

(C2) $\forall P_1, P_2 \in \mathcal{P}, \frac{\partial c(P_1, P_2)}{\partial P_1}, \frac{\partial c(P_1, P_2)}{\partial P_2}$ are continuous and differentiable.

I will make the following assumption on the second derivatives:

(C3) $\frac{\partial^2 c(P_1, P_2)}{\partial P_1^2} > 0, \quad \frac{\partial^2 c(P_1, P_2)}{\partial P_2^2} > 0, \quad \frac{\partial^2 c(P_1, P_2)}{\partial P_1 \partial P_2} < 0$

(C3) (like (C1)) places restrictions on the functional form for $c(., .)$ with certain forms being disallowed, e.g. linear functions of $P_1$ or $P_2$ like $(3P_1 + 2P_2)$, which have zero second derivatives everywhere; separable functions like $(ln(P_1) + P_2^2)$, which have zero cross-derivatives everywhere. As we will see below, this assumption is made for technical reasons and guarantees some properties, including regular optima. More importantly, (C1)-(C3), which I believe are reasonable, imply that

\(^{15}\)This can be shown in the following manner:

Proof. Consider $\exists P_1, P_2 \in \mathcal{P}, P_1 \neq P_2$. From (C1), $c(P_1, P_1) = 0$ and $c(P_1, P_2) > 0$. Since $c(P_1, P_1) \neq c(P_1, P_2)$, $c$ does not depend solely upon $P_1$. The case of $c(., .)$ depending solely upon $P_2$ is similar. \(\square\)
the costs and marginal costs are not independent of the initial and final processes. In economic terms, this can be interpreted as a form of “path dependence” - in two situations characterized by the same initial (final) process, costs and marginal costs can be different if the final (initial) process is different. Essentially, the path of reassignment matters for both costs and marginal costs. $\frac{\partial c(P_1, P_2)}{\partial P_1 \partial P_2} < 0$ can be interpreted as decreasing marginal cost of reassignment, i.e. the marginal cost of reassigning the process decreases in the initial process.

Finally, I assume that initially all the bargaining power is held by the employer, i.e. he/she can make a take-it-or-leave-it offer to the laborer. If there is renegotiation, there are several assumptions that can be made regarding how the surplus can be split. I assume that the employer and the laborer get shares of $\alpha$ and $(1 - \alpha)$, respectively where $0 < \alpha < 1$. As I will show below, this particular way of dividing the surplus is not essential for the results. Finally, it is worthwhile to point out here that in the standard hold-up problem, renegotiation serves to redistribute, but is not Pareto-improving. However, here, there is a potential for Pareto-improvement, as I will discuss below.

2.2 Analysis

I will first solve for the equilibrium allocation by using backward induction. At date $\frac{1}{2}$, let $\theta^*$ denote the true value of $\theta$ and $P_1$ the process that was assigned at date 0. At date $\frac{1}{2}$ when there is renegotiation, let $P_2$ and $w$ denote the (new) process assigned and the wage, respectively. The surplus of the employer is $(y(P_2; \theta^*) - i(P_2) - c(P_1, P_2) - w - (-i(P_1)))$. Note that since the process assigned at date 0 is $P_1$, the employer has a specific investment of $i(P_1)$ at date $\frac{1}{2}$, which will be lost if the laborer walks away. In other words, if the laborer walks away, the employer has a profit of $(-i(P_1))$. The laborer’s surplus is $(w + g(P_2) - u)$. The total surplus
is therefore:

\[ S = [y(P_2; \theta^*) - i(P_2) - c(P_1, P_2) + i(P_1) + g(P_2) - u] \quad (9) \]

During renegotiation, this surplus is maximized by choosing \( P_2 \in \mathcal{P} \) taking \( P_1 \) as given. The first order condition is given by:

\[ \frac{\partial y(P_2; \theta^*)}{\partial P_2} - \frac{\partial i(P_2)}{\partial P_2} - \frac{\partial c(P_1, P_2)}{\partial P_2} + \frac{\partial g(P_2)}{\partial P_2} = 0 \quad (10) \]

Note that the assumptions on \( y, i, u \) and \( c \) guarantee that the second order condition for a regular maximum is satisfied. Let the solution to the above equation be denoted as \( P^*_2(P_1; \theta^*) \). In the analysis below, to make the notation compact, I will suppress the arguments and use \( P^*_2 \). The lemma below derives some useful properties:

**Lemma 1.** \( \frac{\partial P^*_2}{\partial P_1} > 0, \frac{\partial P^*_2}{\partial \theta^*} > 0 \)

**Proof.** Differentiating (10) with respect to \( P_1 \) and rearranging, we get:

\[ \frac{\partial P^*_2}{\partial P_1} = \frac{\partial^2 c(P_1, P^*_2)}{\partial P_1 \partial P_2} \left( \frac{\partial^2 y(P^*_2; \theta^*)}{\partial P^*_2} - \frac{\partial^2 i(P^*_2)}{\partial P^*_2} - \frac{\partial^2 c(P_1, P^*_2)}{\partial P^*_2} + \frac{\partial^2 g(P^*_2)}{\partial P^*_2} \right) \quad (11) \]

\( \frac{\partial P^*_2}{\partial P_1} > 0 \) follows from the second order condition and assumption (C3) (i.e. \( \frac{\partial^2 c(P_1, P_2)}{\partial P_1 \partial P_2} < 0 \)).

Similarly, differentiating (10) with respect to \( \theta^* \) and rearranging, we get:

\[ \frac{\partial P^*_2}{\partial \theta^*} = -\frac{\partial^2 y(P^*_2; \theta^*)}{\partial P^*_2 \partial \theta^*} \left( \frac{\partial^2 y(P^*_2; \theta^*)}{\partial P^*_2} - \frac{\partial^2 i(P^*_2)}{\partial P^*_2} - \frac{\partial^2 c(P_1, P^*_2)}{\partial P^*_2} + \frac{\partial^2 g(P^*_2)}{\partial P^*_2} \right) \quad (12) \]

\( \frac{\partial P^*_2}{\partial \theta^*} > 0 \) follows from \( \frac{\partial^2 y(P^*_2; \theta^*)}{\partial P^*_2 \partial \theta^*} > 0 \) and the second order condition. \( \square \)

Let the optimal surplus from the above problem be denoted \( S^* \). This is derived by substituting for \( P^*_2(P_1; \theta^*) \) in equation (9) above. The profit of the employer after renegotiation is:

\[ \pi(P_1, \theta^*) = \alpha S^* - i(P_1) = \alpha[y(P^*_2; \theta^*) - i(P^*_2) - c(P_1, P^*_2) + i(P_1) + g(P^*_2) - u] - i(P_1) \quad (13) \]
At date 0, the objective of the employer is to choose $P_1$ and $w$ to maximize his/her expected profit, taking into account both the possibility of renegotiation and the participation constraint of the laborer. Note that since the employer does not know the true distribution of $\theta$, he/she uses the signal $(s)$. In other words, the employer’s problem is:

$$\max_{\{P_1, w\}} \int_A \pi(P_1, s) dF_s(s)$$

(14)

$$w + g(P_1) \geq u$$

(15)

$$w \geq 0; P_1 \in \mathcal{P}$$

(16)

Let the solution to this problem be $(\hat{P}_1, w^*)$. Using (10) and (13), the first order condition can be written as:

$$\frac{\partial \int_A \pi(P_1, s) dF_s(s)}{\partial P_1} = -\alpha \int_A \frac{\partial c(P_1, P^*_2)}{\partial P_1} dF_s(s) + (1 - \alpha) \frac{\partial i(P_1)}{\partial P_1} = 0$$

(17)

The above expression has an interesting interpretation. The process chosen at date 0 affects the final allocation through two channels: costs of reassignment and the outside option that the employer has during renegotiation, which in turn depends upon the specific investment made at date 0. The more sophisticated the process assigned at date 0, the higher is the specific investment, and the lower is the outside option, and therefore the profit during renegotiation. Essentially, there is an incentive for the employer to assign a less sophisticated process at date 0 anticipating the hold-up problem. However, assigning a less sophisticated process at date 0 could result in higher costs of reassignment at date $\frac{1}{2}$. Note the importance of the assumption of “path dependence” here. If $\frac{\partial^2 c(P_1, P^*_2)}{\partial P_1 \partial P^*_2} = 0$, then from (10), we can see that the optimal process at date $\frac{1}{2}$ does not depend upon the process assigned at date 0. In this case, the employer will assign the least sophisticated process at date 0, which involves no investment, and there is no hold-up problem.
The overall solution is given by (10) and (17). I will call this the “Equilibrium with Unforeseen Contingencies” and compare it with two different kinds of optimal allocations - “first-best” and “constrained first best”. The former is the best among all allocations, including those with no unforeseen contingencies, i.e. if the true distribution of $\theta$ is known. The latter is the best among all allocations with unforeseen contingencies, i.e. where the true distribution of $\theta$ is unknown, and the signal $(s)$ is used.

I will first characterize the first-best allocation. This can be done in a manner similar to the above. In fact, the problem at date $\frac{1}{2}$ is the same and therefore the first order condition is the same. At date 0, the expected surplus is to be maximized:

$$\max_{\{P_1\}} S_{FB} = \left[ \int_A y(P_2^*; \theta) dF_\theta(\theta) - i(P_2^*) - c(P_1, P_2^*) + g(P_2^*) - u \right]$$

(18)

Note that $P_2^*$ is as defined earlier, from equation (10). Differentiating the above expression with respect to $P_1$ and using (10), we obtain the first order condition:

$$\frac{\partial S_{FB}}{\partial P_1} = - \int_A \frac{\partial c(P_1, P_2^*)}{\partial P_1} dF_\theta(\theta) = 0$$

(19)

Given (C3) (i.e., $\frac{\partial^2 c(P_1, P_2^*)}{\partial P_1^2} > 0$), the second order condition for a regular maximum is satisfied. The optimal solution is given by (10) and (19). Let the process assigned at date 0 be denoted $\tilde{P}_1$.

I will now characterize the constrained first best allocation, which is the best among allocations with unforeseen contingencies. This can be done in a manner similar to the one used to characterize the first-best allocation. The only difference is that at date 0, the optimal process (denoted $\tilde{P}_1$) is obtained by solving (18) with $s$ instead of $\theta$ (since the distribution of $\theta$ is unknown). The expressions analogous
to (18) and (19) are:

$$\max \{ P_1 \} S_{CFB} = \left[ \int_A y(P_2^*; s) dF_s(s) - i(P_2^*) - c(P_1, P_2^*) + g(P_2^*) - \mu \right]$$  \hspace{1cm} (20)

$$\frac{\partial S_{CFB}}{\partial P_1} = - \int_A \frac{\partial c(P_1, P_2^*)}{\partial P_1} dF_s(s) = 0$$  \hspace{1cm} (21)

How does the equilibrium with unforeseen contingencies compare with the first-best and constrained first-best? This is the question that I will address below.

Before going into the details, it is worthwhile to reiterate that the initial process assigned in the equilibrium with unforeseen contingencies, first-best and constrained first-best are denoted as $\hat{P}_1$, $\bar{P}_1$ and $\bar{P}_1$, respectively. The final processes are denoted $P_2^*(\hat{P}_1, \theta^*)$, $P_2^*(\bar{P}_1, \theta^*)$ and $P_2^*(\bar{P}_1, \theta^*)$, respectively. The result below compares the processes assigned under the equilibrium with unforeseen contingencies and constrained first-best:

**Lemma 2.** $\hat{P}_1 < \tilde{P}_1$ and $P_2^*(\hat{P}_1, \theta^*) < P_2^*(\tilde{P}_1, \theta^*)$

**Proof.** From (17) and (21), we can see that $\frac{\partial \int_A \pi(\bar{P}_1, s) dF_s(s)}{\partial P_1} = -(1 - \alpha) \frac{\partial i(\bar{P}_1)}{\partial P_1} < 0$.

Since $\hat{P}_1$ is the optimal solution from (17), $\hat{P}_1 < \tilde{P}_1 \Rightarrow P_2^*(\hat{P}_1, \theta^*) < P_2^*(\tilde{P}_1, \theta^*)$ (from Lemma 1).

Due to the hold-up problem, the date 0 investment in the equilibrium with unforeseen contingencies is less than the same in the constrained first-best case. Given the decreasing marginal cost of reassignment, the process assigned at date $\frac{1}{2}$ (which is what really matters in terms of the process assigned) is less sophisticated compared to the same under constrained first-best. I now perform a similar comparison with the first-best allocation. Before doing so, it is worthwhile to compare (17) and (19), the first order conditions characterizing the equilibrium with unforeseen contingencies and the first-best, respectively. We can see that they yield different solutions, i.e. $\hat{P}_1 \neq \bar{P}_1$ except in a very special case (i.e. when...
\[-\int_A \frac{\partial c(\bar{P}_1, P_2^*)}{\partial P_1} dF_\theta(\theta) = 0\], which I assume does not hold. Unlike in the case of comparison with the constrained first-best, here the comparison of processes assigned is more complicated. From (17), we can see that:
\[
\partial \frac{A}{\pi(P_1, s)} dF_s(s) \frac{\partial}{\partial P_1} = -\frac{c(\bar{P}_1, P_2^*)}{\partial P_1} dF_s(s) + (1 - \alpha) \frac{\partial i(\bar{P}_1)}{\partial P_1} \tag{22}
\]

The process assigned under the equilibrium with unforeseen contingencies is less sophisticated than the same under first-best if and only if:
\[
-(\alpha \frac{A}{\pi(P_1, P^*)} dF_s(s) + (1 - \alpha) \frac{\partial i(\bar{P}_1)}{\partial P_1}) < 0 \tag{23}
\]

Multiplying the first order condition from first-best (equation (19)) by \(\alpha\) and using it in (23), we get:
\[
\alpha \left(\int_A -\frac{c(\bar{P}_1, P^*)}{\partial P_1} dF_s(s) - \int_A -\frac{c(\bar{P}_1, P_2^*)}{\partial P_1} dF_\theta(\theta)\right) - (1 - \alpha) \frac{\partial i(\bar{P}_1)}{\partial P_1} < 0 \tag{24}
\]

The above expression can be written in a compact manner and interpreted better by rewriting it in the following manner:
\[
\alpha \left(\int_A G(s; \bar{P}_1, P_2^*) dF_s(s) - \int_A G(\theta; \bar{P}_1, P_2^*) dF_\theta(\theta) - (1 - \alpha) \frac{\partial i(\bar{P}_1)}{\partial P_1}\right) < 0 \tag{25}
\]

where, \(G(x; P_1, P_2) = -\frac{\partial c(P_1, P_2^*)}{\partial P_1}, x = s, \theta\).

The above expression has an interesting interpretation. Essentially, there are two inefficiencies at play, which are represented respectively by the two terms: (i) unforeseen contingencies, which could lead the employer to assign either a less or more sophisticated process, and (ii) the hold-up problem, which leads the employer to assign a less sophisticated process. If both these factors work in the same direction, then the process assigned under the equilibrium with unforeseen contingencies is less sophisticated. On the contrary, if the first term is positive, but not large enough, then although these factors work in the opposite direction, the second factor dominates. We can derive a sufficiency condition based upon (i) and (ii). First,
a sufficient condition is that the signal systematically "underestimates" the true fit, so that the process that is assigned initially is less sophisticated, which results in a less sophisticated process being assigned during renegotiation. Here, both (i) and (ii) work in the same direction. Second, the degree of incompleteness is not "large," so that even if the first factor leads the employer to assign a more sophisticated process, the second factor dominates. The lemma below presents this result.

**Lemma 3.** $P^*_2(\hat{P}_1, \theta^*) \neq P^*_2(\bar{P}_1, \theta^*)$. A sufficient condition for $\hat{P}_1 < \bar{P}_1$ and $P^*_2(\hat{P}_1, \theta^*) < P^*_2(\bar{P}_1, \theta^*)$ is either: (S1) $\theta$ dominates $s$ stochastically at first order, or (S2) $\forall P_1, P_2 \in \mathcal{P}, D(G(.)) \leq \frac{(1-\alpha)}{\alpha} \frac{\partial_i(P)}{\partial P_1}$

Proof. (i) $P^*_2(\hat{P}_1, \theta^*) \neq P^*_2(\bar{P}_1, \theta^*)$ follows from $\hat{P}_1 \neq \bar{P}_1$ and lemma 1. (ii) $\frac{\partial G}{\partial x} = -\frac{\partial^2 c(P_1, P^*_2(P_1, x))}{\partial P_1 \partial P_2} \frac{\partial P^*_2(P_1, x)}{\partial x} > 0$, given lemma 1 and assumption (C3) $(\frac{\partial^2 c(P_1, P^*_2(P_1, x))}{\partial P_1 \partial P_2} < 0)$. Since $G$ is increasing in $x$, if (S1) is satisfied, $\int_A G(s; P_1, P_2) dF_s(s) \leq \int_A G(\theta; P_1, P_2) dF_\theta(\theta)$, which implies that (25) is satisfied and therefore $\hat{P}_1 < \bar{P}_1 \Rightarrow P^*(\hat{P}_1, \theta^*) < P^*(\bar{P}_1, \theta^*)$ (from Lemma 1). (iii) If $\int_A G(s; \bar{P}_1, P^*_2) dF_s(s) - \int_A G(\theta; \bar{P}_1, P^*_2) dF_\theta(\theta) < 0$, then the result follows from above. Otherwise, if (S2) is satisfied, the result follows from: $D(G(.)) = |\int_A G(s; \bar{P}_1, P^*_2) dF_s(s) - \int_A G(\theta; \bar{P}_1, P^*_2) dF_\theta(\theta)| \leq \frac{(1-\alpha)}{\alpha} \frac{\partial_i(P)}{\partial P_1} < \frac{(1-\alpha)}{\alpha} \frac{\partial_i(\bar{P}_1)}{\partial P_1}$. Note that $\frac{\partial^2 c(P)}{\partial P_2} > 0$. □

Note that when $\theta$ dominates $s$ stochastically, both employer and laborer “underestimate” the true fit, and therefore underestimate the returns from the project.\(^{16}\)

The proposition below summarizes the main implication of the above two results.

**Proposition 4.** The allocation under the equilibrium with unforeseen contingencies is neither first-best nor constrained first-best.

Proof. Follows from Lemma 2 and Lemma 3. □

\(^{16}\)Note that $y(P; \theta)$ is increasing in $\theta$. 19
3 Discussion and Conclusions

In the above analysis, I have formalized the idea from classical and radical political economy that given employers’ desire to control the production process, the allocation of tasks and the adoption of technology is sub-optimal. I have relied on incomplete contracts of a particular kind, i.e. where contingencies are unforeseen or indescribable. The presence of these contingencies leads to the possibility of Pareto-improving renegotiation, but also to a hold-up problem. I used two different notions of optimality, and showed that the allocation is inferior to all allocations (first-best) and all allocations where contingencies are unforeseen (constrained first-best). What is crucial here is that there are costs of renegotiation and “path dependence” - the process (tasks and technology) that is initially assigned affects the costs of reassignment at the time of renegotiation.

Although I have focused upon unforeseen contingencies that affect the output (i.e. through the fit), the results are applicable to other kinds of unforeseen contingencies, e.g. those that affect the preferences of the laborer. Suppose the utility function of the laborer is given by: \( u = w + g(P; \theta) \) where \( \theta \) is a parameter that influences the satisfaction of the laborer from a particular process, and whose distribution is unknown. The analysis can then be performed in a manner similar to the above, and it can be shown that under certain conditions, the resulting allocation is neither first-best nor constrained first-best.

I have relied on the hold-up problem and the idea of power as dependence. However, the analysis in this paper is different from standard treatment of the hold-up problem (e.g as in Denrell (2000)). Consider the following situation that can be used to illustrate the standard hold-up problem: A seller can deliver to a buyer a product, after enhancing its value by investing. If the seller invests \( i \), he/she incurs
a cost of \( c(i) \) \((c(0) = 0, c'(i) > 0)\) and the value of the product to the buyer is \( v(i) \) \((v'(i) > 0)\). The amount of investment is non-contractible. It is easy to show that if the possibility of renegotiation exists, given the hold-up problem, the investment is less than first best. The present analysis is different from this standard situation in several ways. First, as shown above, the results obtained in the present analysis are different. In particular, we have the role of path-dependence and the result that the degree of incompleteness affects the extent to which the allocation is inefficient. Second, in standard treatments of the hold-up problem, renegotiation only serves to redistribute, whereas here it can also be Pareto improving. As a result, certain mechanisms that can solve the hold-up problem are ineffective here.\(^{17}\) For example, in the standard case, if the employer and laborer write down a “good faith” clause which commits them to not renegotiate, and which can be enforced by a neutral third-party, this mechanism will solve the hold-up problem (i.e. deliver first-best). Since, the seller is assured that there will be no renegotiation, he/she will put in the first-best level of investment. Here, commitment to not renegotiate will not deliver the first-best or even constrained first-best because although it solves the hold-up problem, it also prevents the possibility of Pareto-improvements.

Another mechanism that could work in the standard case is a “default option” - in case trade does not happen, then the seller is entitled to some (“default”) payment that is agreed upon by the buyer and the seller and that can be enforced by a neutral third party. This mechanism will deliver first-best since the default payment can be set equal to the profit that the seller obtains from first-best investment, if there is no hold-up problem (i.e. no renegotiation). The seller will put in the first-best investment because even if the buyer forces renegotiation, the seller is guaranteed the default payment (which is set in the above-mentioned manner).

\(^{17}\)For a discussion of some of these, see Bolton and Dewatripont (2005), section 12.3.
In the current model, this mechanism will not deliver first-best or even constrained first-best. The reason is that ex-ante, the profit from first-best or constrained first-best investment is not known so that the default payment cannot be set as above.

The thrust of the above results is that when unforeseen contingencies are introduced, the implications for efficiency are more complex compared to the standard model. On the one hand, the allocation is inefficient only if path dependence is satisfied. On the other hand, when path dependence is satisfied, not only is the allocation inefficient, but also the inefficiency cannot be prevented by using mechanisms that work in the standard case.

Finally, it is worthwhile to look at the results in the context of the literature on unforeseen contingencies. As mentioned in the introduction, this literature has been somewhat contested with several relevance and irrelevance results. It has also raised some questions that go to the foundations of what constitutes an incomplete contract. The treatment in this literature is fairly abstract. Hence, this paper can be seen as contributing to this literature in two ways: First, it provides an instance where unforeseen contingencies do and do not matter in a concrete setting. Second (and related to the above), it links this literature to older and wider debates within economics and other social sciences.

References


**Grout, Paul.** 1984. “Investment and Wages in the Absence of Binding Contracts:


Figure 1: Sequence of Events

(0) __________________  (1/2) __________________ (1)

Contract is signed   \( \theta \) is revealed   Contract is executed
using signal(s) and    and renegotiation   and returns are
process is assigned    occurs                 obtained