VERTICAL INTEGRATION VERSUS OUTSOURCING
IN INDUSTRY EQUILIBRIUM

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Abstract
We study the determinants of the extent of in-house vertical integration and of outsourcing in foreign countries. Potential suppliers must make a relationship-specific investment in order to serve each prospective customer. Such investments are governed by imperfect contracts. A final-good producer can manufacture components for it, but the per-unit cost is higher than for specialized suppliers. We consider how the size of the cost differential, the trade costs of components, the relative costs of searching in south country for final producers, the relative cost of customizing inputs affect the organization of industry production.

Key Words: Outsourcing, Vertical Integration, Customization, Search Cost
1. Introduction

We live in an age of outsourcing. Firms seem to outsource more and more activities, ranging from product design to assembly, from research and development to marketing, distribution and after-sales service. The rising integration of world markets brought a disintegration of the production process, in which manufacture or services activities done abroad are combined with those performed in home country. For example, Feenstra (1998) have described the production process of Nike shoes and clothing:

“About 75,000 people are employed in Asia in the production of shoes and clothing for Nike, though only a few hundred of these are actually employees of the company. The rest are employed in factories that have some contractual arrangement with Nike, possibly run by third parties, such as South Korean entrepreneurs. Along with this massive, albeit indirect, workforce in Asia, Nike has some 2,500 employees in the United States. The worldwide sales of Nike shoes generated profits of $360 million in 1993.”

We assume that a firm producing final goods for consumers needs components as inputs. Generally speaking, there are four kinds of organization forms in a single industry, a final goods producer can select in-house vertical integration, it can outsource the inputs in the home country, also it might choose to produce them in a subsidiary that is located in a foreign country, or import them from a foreign supplier. Since it is complicated to consider all of the four forms simultaneously, most studies focus on the trade-offs between more limited modes of organization. Grossman and Helpman (2000a) studied the relationship between outsourcing in the home country and outsourcing from abroad. What’s more, Grossman and Helpman (2000b) studied the trade-off between outsourcing and in-house vertical specialization in a closed economy. Also, Grossman and Helpman (2003) examined the trade-off between FDI and outsourcing from abroad. Here we try to illustrate the relationship between in-house vertical integration and outsourcing from abroad, which seems to be the trend of the world economy. Based on the general equilibrium model of production and trade developed by Grossman and Helpman, this paper tries to examine the organization modes of production.

We assume that producers of final consumer goods are located in the North, and some components producers are located in the South. The wage in the South is lower than that in the North, but there are some kinds of trade costs for inputs trade from the

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1 There are some differences between our model and that given in Grossman and Helpman (2002b). There final producers choose to outsource either in the south or in the north, here final producers choose the modes of organization, outsourcing from abroad or vertical integration.
South to the North, such as tariffs. Final producers choosing the mode of outsourcing must search for suppliers in the South, and those that choose the mode of vertical integration must produce input components by themselves in the North.

Based on the basic model developed in section 2 and 3, we examine several potential determinants of the scale ratio of vertical integration to foreign outsourcing in section 4. The industry equilibrium takes account of the effects of entry and exit of final and intermediate good producers on the profitability of every firm. And we consider labor supply to be infinitely inelastic. We find that, a higher tariff level for input trade tends to increase the quantity of firms engaged in vertical integration, and to decrease the quantity of outsourcing final producers; a more advanced technology for outsourcing will lessen the scale of vertical integration and increase that of outsourcing from aboard. An increase in the labor endowment of the south has a similar effect as technology for outsourcing. At last, a more efficient productivity technology for those vertical integrated firms in the north tends to increase the relative size of the group of firms that produce their input component by themselves.

2. The Model

Consider a world economy with two countries, north and south, and two industries. Firms in either country can produce a homogeneous consumer good \( z \) with one unit of local labor per unit of output. Only firms in the north can design and assemble varieties of a differentiated final good \( y \) for consumers, firms in the south do not know the relevant technologies to do so. Both firms in the north and south possess the knowledge of how to produce intermediate goods. The components are necessary inputs to produce good \( y \).

We have just mentioned that the varieties of final good \( y \) are differentiated. Firstly, it means that consumers may regard the different products as imperfect substitutes; secondly, it means that different final good require different components in their production. On the demand side, we can thus use a CES sub-utility function. On the supply side, we associate each kind of final good with a point on the circumference of a unit circle, and the “location” of a final good means the specifications of the input needed for production.

We assume that consumers in the north and south have the same preference; their identical utility function has the form of equation (1)

\[
 u = z^{1-\beta} \left[ \int_0^1 \int_{l_0}^{l_n(i)} y(j, l)^{\alpha} djidl \right]^{\beta \over \alpha}, 0 < \alpha, \beta < 1
\]
Where $z$ is consumption of the homogenous consumer good $z$, $y(j,l)$ is consumption of the $j$-th variety located at point $l$ on the unit circle. We assume that a continuum of goods are located at each point on the circle, and equation (1) means that consumers consider varieties of final good $y$ as differentiated even they lies on the same point. $n(l)$ measures the varieties of final good $y$ that require an intermediate input at location $l$. As usual, $\beta$ gives the spending share that consumers will devote to the homogenous good $z$ and $\varepsilon = 1/(1-\alpha)$ means the elasticity of substitution between any pair of the varieties of final good $y$.

Now we can turn to the supply side, the production of any variety of good $y$ requires a fixed investment in product design and one unit of the customized input per unit of output. Producers for final good $y$ in the north can select the modes of in-house vertical integration and outsourcing from abroad. For those choosing in-house vertical integration, the fixed cost is $f_v$ units of northern labor, which probably includes resources needed to enter the market, those needed to design a product and those for corporate governance. For firms choosing outsourcing from abroad, the fixed cost is made up of two factors, the first is input requirement similar to those for a vertically integrated firm, which costs $f_n$ units of northern labor, the second is the search cost for them to seek suppliers in the south for their specialized inputs, and it may cost $f_s$ units of northern labor, we assume that by bearing this search cost, a firm can ascertain the expertise of all suppliers active in the south and identify the one whose expertise is closest to its own input needs. For simply, we assume that $f_v = f_n + f_s$.

There are also component suppliers in the south, and the fixed cost for them to entry is $f_m$ units of southern labor, a supplier’s expertise is represented by a point on the unit circle. The suppliers lie equally around the circle, for simply, we assume that the finite number of input suppliers in the south as a continuous variable.

A vertically integrated firm in industry requires $\lambda \geq 1$ units of north labor to produce a unit of components for further production, the possibility that a vertically integrated firm needs more labor to produce one unit of input than those suppliers in the south reflects the fact that its activities are not highly specialized and the cost for

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2 Since there is a continuum of differentiated final goods, the fixed cost of designing a single product is infinitesimally small. Of course, the total units of labor used in designing a positive measure of such goods are finite.
managing a larger operation is higher. The wages in the south and north are \( w^S \) and \( w^N \) respectively, and we can set that \( w^N > w^S \), so we have \( \omega = w^N / w^S > 1 \).

We assume that tariffs for trade of homogenous good \( z \) and varieties of final good \( y \) are both zero, but when it comes to trade of components, the tariff is not zero. Exactly, the tariff level is \( t \) per unit of components. So the entire world output of the homogenous good \( z \) is produced in the south since the relatively lower wage there. We assume that there are \( n_1 \) vertically integrated final good producers for good \( y \) and \( n_2 \) final good producers which select outsourcing from abroad in the north, when it comes to input suppliers in the south, we assume that there are \( m^S \) such firms.

Once a final good producer decides to select the mode of vertical integration, it must go to the southern market to conduct its search. It only knows the aggregate number of such suppliers and lacks the information of the expertise of all of the various potential suppliers before conducting search. It may regard all equi-spaced configurations of suppliers in the south as equally likely, so at the time of searching in the south, a final good producer knows that the nearest supplier will be at a random distance \( x \) ranging from 0 to \( 1/2 m^S \).

A firm that specialized in producing intermediated goods in the south must develop a prototype before it can produce the customized inputs required by a certain final producer. This cost varies according to the distance between the location of the supplier’s expertise and that of the final good producer’s input requirement. We assume that if the distance between an inputs supplier and a final producer, which receives this supplier’s products, is \( x \), the corresponding cost will be exactly \( w^S \mu^S x \).

After developing a certain prototype for a certain final producer in the north, the component producer in the south can produce customized components for its partner at constant marginal cost, with one unit of southern labor needed for per unit of output.

2.1. Bargaining and contracting
For now we assume that none of the investment contract is made between the component suppliers and final producers, the result of this assumption is that the suppliers must be willing to undertake the investment and bear the total costs of
customization themselves in anticipation of an order contract, which may be negotiated only after a suitable prototype has been built.\(^3\)

Let \( s^o \) denote the total profits the parties will share if a negotiation is made between a component supplier and a component buyer, and each party will obtain an equal sharing of \( s^o \). The component supplier will get \( s^o / 2 \) if it chooses to invest \( w^s \mu^s x \) in the prototype, where \( x \) measures the distance between the final producer’s requirement and the closest supplier’s expertise. The natural result is that the component supplier is willing to take the investment if and only if \( w^s \mu^s x \leq s^o / 2 \).

A final good producer selecting the mode of outsourcing will have no choice but to exit the industry if it finds that the nearest component supplier lies at a distance greater than \( s^o / 2w^s \mu^s \). If we define \( r^s \) as the greatest distance between any producers that remain active after having searched for a partner in the south and its corresponding supplier, we can have the following equation

\[
r^s = \min \left\{ \frac{s^o}{2w^s \mu^s}, \frac{1}{2m^s} \right\}
\]

(2)

From the utility function (1), we can get that the demand for the j-th variety of good y at location \( l \) is given by

\[
y(j, l) = Ap(j, l)^{-\varepsilon}
\]

(3)

Here \( p(j, l) \) is the price charged for the j-th variety of good y at location \( l \) and

\[
A = \frac{\beta \sum_i E^i}{\int_0^1 \int_0^{n(i)} p(j, l)^{1-\varepsilon} \, dj \, dl}
\]

(4)

Where \( E^i \) means the total spending of consumers in country i, for \( i = N, S \). The demand function has a constant demand elasticity, which means that final good producers can maximize profits by fixed mark-up pricing. For those final producers outsourcing from abroad, the marginal cost is \( w^s + t \), so they will set the price of

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\(^3\) We assume that a final producer’s input requirements are unique, and different from those of other final producers even they lie at the same point on the unit cycle. And final producers only use components that exactly fit their requirements. Those assumptions simplify the model without significantly affecting the results.
their differentiated good \( y \) at the level 
\[ p^s = \frac{w^S + I}{\alpha}, \]
and the maximal joint operating profits are
\[ s^o = (1 - \alpha)A(\frac{w^S + I}{\alpha})^{1-\varepsilon} \]  
(5)
The corresponding quantity of output is
\[ y^S = A(\frac{w^S + I}{\alpha})^{-\varepsilon} \]  
(6)
The total payment from a final good producer to its partner is
\[ \frac{s^o + w^S y^S + I y^S}{2} = \frac{1 + \alpha}{2} A(\frac{w^S + I}{\alpha})^{1-\varepsilon} \]  
(7)
For those final good producers choosing the mode of vertical integration in the north, the marginal cost of production is \( \lambda w^N \), so the corresponding final good price set by vertically integrated firms is 
\[ p^v = \frac{\lambda w^N}{\alpha}, \]
the maximal profit of a vertically integrated firm is
\[ s^v = (1 - \alpha)A(\frac{\lambda w^N}{\alpha})^{1-\varepsilon} \]  
(8)
The corresponding output of a single vertically integrated producer is
\[ y^v = A(\frac{\lambda w^N}{\alpha})^{-\varepsilon} \]  
(9)

2.2. Search
For those final producers choosing the mode of outsourcing from abroad, they should search for their partners in the south in order to get specific components for their production. We have just mentioned that a final producer from the north will find its partner at a random distance \( x \), which is uniformly distributed on the interval \([0, 1/2m^S]\). If a final producer finds its partner at a distance greater than \( r^S \), the component supplier will refuse to undertake the investment in customizing the intermediates, so both the final producer and the component supplier will get zero profits. On the other hand, if a final producer finds a partner at a distance closer than

\[ \text{The payment is such that the input supplier’s reward net of manufacturing costs is half of the joint profits.} \]
the supplier may be willing to customize, in this event, both the final producer and its partner will earn \( \frac{s^o}{2} \). Since the probability of customization is \( 2r^s m^s \) and the probability of no customization is \( 1 - 2r^s m^s \), the expected operating profits of a final producer choosing the mode of outsourcing from the south are

\[
\pi_o = r^s m^s s^o
\]  

(10)

### 2.3. Free entry and market clearing

The assumption of free entry means that all entrants earn zero expected profits in equilibrium. The free entry condition for a final producer looking for its partner in the south is

\[
\pi_o = w^s (f_n + f_s)
\]  

(11)

A component supplier in the south may serve a measure \( 2n_z r^s \) of final good producers choosing the mode of outsourcing, and those final producers lie uniformly at distances ranging from 0 to \( r^s \) in each direction from the point where the supplier’s expertise is located. Since a component supplier will get \( s^o / 2 - w^s \mu^s x \) from its partnership with a final producer whose component requirement is at a distance \( x \) from its expertise, the operating profits for an input producer in the south are

\[
\pi_m = 2n_z \int_0^{r^s} \left( \frac{s^o}{2} - w^s \mu^s x \right) dx = r^s n_z \left( s^o - w^s \mu^s r^s \right)
\]  

(12)

The free entry condition for a component supplier in the south is

\[
\pi_m = w^s f_m^s
\]  

(13)

When it comes to the vertically integrated producers in the north, the free entry condition takes the form of equation (14)

\[
\pi_n = w^s f_v^s
\]  

(14)

Now we turn to the labor-market clearing condition in both the northern market and the southern market. Firstly, we consider the southern labor market, since the wage in the north is higher \( w^N > w^s \), the entire output of homogenous good \( z \) is produced in the south. The fact that net profits are zero for all firms ensures that aggregate spending equals aggregate income for consumers in each country, which

\footnote{The component producers also choose their locations. We assume that this choice is made with rational expectations. It is a dominant strategy for each firm to lie at a point mid-way between the expected locations of the two most-distantly-spaced adjacent producers of components.}
implies \( E^i = w^i L^i \), where \( L^i \) is the labor supply in country \( i \). A fraction \( 1 - \beta \) of consumers’ spending is devoted to homogeneous goods \( z \), the price of which is \( w^s \). Moreover, we assume that the northern government spends all of its tariff income on good \( z \), since each component supplier in the south will serve a measure \( 2n_z r^s \) of final good producers, the total quantity of components trade is \( 2m^s n_z r^s y^s \), and the total tariff income of the northern government is \( 2tm^s n_z r^s y^s \). Under those assumptions, we can see that in equilibrium, the south will employ

\[
\frac{(1 - \beta)(w^s L^s + w^N L^N) + 2tm^s n_z r^s y^s}{w^s} \text{ units of southern labor to produce good } z.
\]

Those input producers in south also use labor to enter the component market, to invest in customization and to produce components, which are necessary inputs for those final goods producers in north. Since there are a total number of \( m^s \) component suppliers and each requires \( f^s_m \) units of southern labor for investment in expertise and equipment, the total units of labor required for this kind of investment are \( m^s f^s_m \). The second type of investment is customization, which requires \( \mu^s x \) units of labor for a final-good producer whose needs in expertise are a distance \( x \) from the expertise of the component producer. Each of the \( m^s \) component producers undertakes such kind of investment for all those final-good producers that search in the south and lie within a distance of \( r^s \) to its right or to its left. Because there are \( n_z \) final-producers in south searching for their partners from north, the total number of southern labor needed for developing prototypes is \( 2\mu^s m^s n_z \int_0^{r^s} x dx = \mu^s m^s n_z (r^s)^2 \). Finally, the density \( n_z \) of northern final good producers searching in south results in a total number of \( 2m^s r^s n_z \) of bilateral relationships, each of which means a quantity of \( y^s \) units of southern labor to produce components as inputs, so those kind of production requires a total number of \( 2m^s r^s n_z y^s \) units of southern labor. From above, we can get the equation expressing the equilibrium of the southern labor market.
\[
(1 - \beta)(w^S L^S + w^N L^N) + \frac{2m^S n_2 r^S y^S}{w^S} + m^S f^S_m + \mu^S m^S n_2 (r^S)^2 + 2m^S r^S n_2 y^S = L^S \quad (15)
\]

Now we turn to the northern labor market, which has two kinds of final-good producers, those choosing the mode of vertical integration and those choosing the mode of outsourcing from abroad. The former producers use northern labor in two ways, firstly, they should devote labor to investment in expertise and equipment in order to enter the production stage. Secondly, some units of labor should be used as factors in the production of input components, in this stage, \( \lambda \) units of northern labor are needed for one unit of component, and then final producers can manufacture final goods using components as input without any other labor inputs any more. In the entry stage, each firm needs \( f_v \) units of labor as investment and there are \( n_i \) such final-good producers choosing the mode of vertical integration in all, so the total units of northern labor needed for investment in expertise and equipments are \( n_i f_v \). In the production stage, since each final producer will produce a quantity \( y^v = A(\frac{\lambda W^N}{\alpha})^{-\varepsilon} \) of final outputs, and each unit of final outputs needs one unit of components as input, moreover, each unit of components needs \( \lambda \) units of northern labor as input, the total units of labor needed in the production stage for the final producers choosing the mode of vertical integration are \( n_i \lambda A(\frac{\lambda W^N}{\alpha})^{-\varepsilon} \). When it comes to those producers choosing the mode of outsourcing from abroad, they also use northern labor in two ways, firstly, they need labor for investment in equipment like those vertically integrated firms, each should devote \( f_i \) units of labor for this purpose. Secondly, they should pay some search cost to ascertain the expertise of all suppliers active in the south, and it may cost \( f_i \) units of northern labor for each of final producers. In total, \( (f_a + f_i) n_2 \) units of northern labor are needed for those vertically integrated final producers. From the above analysis, we can see that the labor-market clearing condition in the north is given by

\[
(f_a + f_i) n_2 + n_i f_v + n_i \lambda A(\frac{\lambda W^N}{\alpha})^{-\varepsilon} = L^N
\]

(16)

This completes the description of the model.
3. Solving for the equilibrium

To gain an understanding of the workings of the model, we focus on the key general equilibrium. It is possible that there are so many component suppliers in the southern market that each final producer searching in the south can find its partner, this requires that $m^S \geq w^S \mu^S / s^o$, but here we only consider the exact type of equilibrium, in which outsourcing takes place in the south and some final-good producers choosing the mode of outsourcing from north can not find suppliers with expertise sufficiently close to their needs for a supply relationship to be consummated after searching in the southern market.

In this case, we can see that equation (2) implies that the greatest distance between an active final producer and its corresponding components supplier is given by

$$r^S = \frac{s^o}{2w^S \mu^S}$$

(17)

The free-entry conditions (12) together with (13) for those components suppliers in the southern market mean that

$$r^S n^s (s^o - w^S \mu^S r^S) = w^S f_m^S$$

(18)

Substituting (17) and (18) into the south’s labor-market clearing condition (15) gives

$$(1 - \beta)(\omega L^N + L^S) + 2 \frac{1 + \alpha}{1 - \alpha} m^S f_m^S = L^S$$

(19)

In equation (19), the first term on the L.H.S. represents part of the labor used in the south in producing in the homogeneous good $z$, which is consumed by residents from both south and north, while the second term measures part of the labor used in producing the homogeneous good $z$, which is consumed by the northern government, and labor used in all activities by component suppliers.

Next, we can substitute equations (5), (8), (10), (11), (12), (13) and (14) into equation (16) to write the following equation which describes the north’s labor-market clearing condition

$$\frac{2m^S}{\omega} f_m^S + \frac{f_m n_n}{1 - \alpha} = L^N$$

(20)

We can see that the equation (19) involves $\omega$ and $m^S$, while the equation (20) involves $\omega$, $m^S$ and $n_n$. But the relative wage, which is expressed as $\omega$, can be
solved as a function of \( \tau \) and \( m^S \) using the intuition that at the stage of entry, a final-good producer must choose which mode of organization it should choose, vertical integration or outsourcing from abroad. Both modes of organization must have relatively equal net profits in the end. Substituting equation (17) into equation (10), together with equations (11) and (14), we can get another statement of the equal-profit condition, which can be expressed as

\[
\alpha \omega = \left( \frac{2\alpha L^N}{m^S S}\right)^{1-\alpha} \left( \frac{1+\tau}{\lambda}\right)^{2\alpha}.
\]

The above equation illustrates that for both modes of organization to be equally profitable, the relative wage must be adjusted to align with the costs of customization, the total number of component suppliers, the relative tariff level and the efficiency of production of vertically integrated final-good producers. The relatively more costly it is for component suppliers to customize components in the south, the more profitable it will be for a final producer to choose the mode of vertical integration, to offset this advantage, the relative wage must be higher in the north. The “thicker” is the market for components in the south, which can be expressed by a larger \( m^S \), the more profitable it will be to select the mode of vertical integration and search in the south for component suppliers, therefore the relative wage must be smaller. If the tariff level becomes much higher, it will be more profitable to select the mode of vertical integration holding other variables constant, to offset this effect, the relative wage rate must be higher. In the end, the more efficient it is to run a company producing final goods in the mode of vertical integration, which can be expressed by a smaller \( \lambda \), the more profitable it will be to select the mode of vertical integration, and to offset this effect, the relative wage rate must be higher.

Combining (19) with (21) yields the following equation in terms of \( \tau \) and \( m^S \).

\[
(1-\beta)\left( \frac{2\alpha L^N}{m^S S}\right)^{1-\alpha} \left( \frac{1+\tau}{\lambda}\right)^{2\alpha} + 2 \frac{1+\alpha}{1-\alpha} m^S f_m^S = \beta L^S.
\]

We can look the variable \( m^S \), which means the “thick” environment of the south market as a function of tariff level \( \tau \). From the above equation, there may be several solutions corresponding to a given \( m^S \). We will analyze those cases in the following graph.

Similarly, combining (19) with (20) yields the following equation in terms of \( \tau \),
In this equation, we can set $\tau$ as an exogenous variable while $\mu^S$ and $n_1$ as endogenous variables, once the northern government set a certain tariff level $\tau$, we can get a relationship between the variables $\mu^S$ and $n_1$. We can see that given $\tau$, the more component suppliers there are in the south market, the less final-good producers choosing the mode of vertical integration in the north given other conditions constant.

The linear relationship between $\omega$ and $m^S$ in equation (19) can be illustrated in the following figure as SS curve, similarly, the nonlinear relationship between $\omega$ and $m^S$ in equation (21) can be illustrated as WW curve.

If we have a look at SS curve, it is evident from (19) that the point $(m_{max}^S, 0)$, where $m_{max}^S = \beta L^S (1-\alpha)/2(1+\alpha)f_m^S$ lies on the curve. It follows that the number of component suppliers in the south is declining with the increase of the relative wage. To understand the economics behind the shape of the SS curve, we can analyze the demand and supply sides of the southern labor market, as the number of component suppliers increases, so does their demand for labor in the south, at a given level of labor supply, the relative wage rate will increase in the south, which is the same as the relative wage decreases in the north.

Figure 1: equilibrium curves
The relative wage reaches its maximum value, which is \( \frac{\beta L^S}{(1-\beta)L^N} \), when there are no component suppliers in the south. And because that the existence of an equilibrium with production of homogeneous goods concentrated in the south requires \( \omega > 1 \), it follows that \( \frac{\beta L^S}{(1-\beta)L^N} > 1 \) is a necessary condition for such kind of an equilibrium we are discussing.

Now we can turn to the shape of WW curve, which has a similar argument. From equation (21), we can see that the relative wage declines with the increase of the total number of component suppliers in the south, and that the relative wage is a concave function of \( m^S \). The economics behind this shape of the WW curve lies in that as the number of component suppliers in the south increases, it will be more profitable for those final-good producers choosing the mode of outsourcing from abroad, in order to offset this effect, the relative wage rate must declines in the north.

If we depict portions of the SS and WW curves from Figure 1 into the same figure together, we can get the following Figure 2.

The dotted line through the point \((0, 1)\) parallel to the horizontal line represents the restricted condition \( \omega = 1 \), we are only interested in the area above this line (which has \( \omega > 1 \)). We can see that there are two equilibria, \( E_1 \) and \( E_2 \), each characterized by active outsourcing in the south \( (m^S > 0) \). Since we have assumed that the equilibrium number of component suppliers is not large enough to ensure every final-good producer choosing the mode of outsourcing from abroad to find its partner in the south, which are willing to incur the relationship-specific investment, we should find some combined conditions for this assumption to be true in our figure.

![Figure 2: equilibrium with binding investment constraints](image-url)
According to equation (2), the investment constraint binds in the south when \( m^s < \frac{w^s \mu^s}{s^o} \), together with equations (10) and (17), we can see that this inequality holds if and only if \( m^s < \frac{\mu^s}{2\omega(f_n + f_s)} \), so the equilibrium points \( E_1 \) and \( E_2 \) satisfy these conditions when \( f_n + f_s \) is small enough. Therefore, the equilibrium points \( E_1 \) and \( E_2 \) fall in the relevant region when the entry and search costs of those final-good producers choosing the mode of outsourcing are small enough.

When there are several equilibria in the same figure, we can not stop wondering which ones are stable. A stability analysis is reported in Figure 2. We assume that the entry and exit of the three kinds of firms—those final producers choosing the mode of vertical integration, those choosing the mode of outsourcing from abroad in the north and the component suppliers in the south—is determined by the corresponding profit opportunities. If profits net of entry costs are positive for a certain kind of producers, more firms of the exact type will enter. Similarly, if profits are negative, firms choose to exit.

Now we consider the entry and exit dynamics described by the arrows in Figure 1, later we will analyze the combined dynamics in Figure 2. Firstly, we focus on a point on the SS curve in Figure 1, suppose that the relative wage rate declines a little, so it will be less profitable for final producers to choose the mode of outsourcing from abroad holding other factors constant, to offset this negative effect and maintain the equilibrium, the total number of component suppliers in the south should increase. We indicate the profit opportunity, which causes entry of component producers in the south, by a rightward horizontal arrow for those points below the SS curve in Figure 1. A similar analysis can be applied to the points above the SS curve, which indicates a leftward horizontal arrow.

We can use a similar method to analyze the dynamic changes of wage rate in Figure 1. From any point on the WW curve, a slight decrease in the number of component suppliers will lessen the profitability of those final producers choosing the mode of outsourcing from abroad, so final producers tend to switch form outsourcing to vertical integration, which may increase the relative wage rate. Through the above analysis, we can see that there is a upward vertical arrow for those points left to the WW curve, and for those right to the WW curve, we have a downward vertical arrow.

Using the dynamic arrows depicted in Figure 1, we obtain the combined dynamics in Figure 2. From the directions of those dynamic arrows, we find that the
equilibrium labeled $E_1$ is stable while the equilibrium labeled $E_2$ is unstable. For such a stable equilibrium to exist, we need two conditions. And in what follows, we focus on economies that satisfy these conditions.

**Condition 1:** the SS curve must have intersections with the WW curve, which can be represented by the following inequality

$$
\left(\frac{(1-\alpha)^2}{4(1+\alpha)f^S_m}\right)^{\frac{1-\alpha}{1+\alpha}} \left(\beta L^S\right)^{\frac{2}{1+\alpha}} - \left(\frac{1-\alpha}{2}\right)^{\frac{\alpha-1}{1+\alpha}} (1+\alpha)^{\frac{\alpha-1}{1+\alpha}} \left(\beta L^S\right)^{\frac{2}{1+\alpha}} \left(f^S_m\right)^{\frac{1-\alpha}{1+\alpha}} > (1-\beta)(2\mu^S)^{\frac{1-\alpha}{1+\alpha}} \left(\frac{1+\tau}{\lambda}\right)^{\frac{2\alpha}{1+\alpha}}
$$

(24)

Proof: combining (19) and (21), we can get (22), which can be transfigured into the following equation

$$(m^S)^{\frac{1-\alpha}{1+\alpha}} \beta L^S - \frac{2(1+\alpha)}{1-\alpha} (m^S)^{\frac{2}{1+\alpha}} f^S_m = (1-\beta)(2\mu^S)^{\frac{1-\alpha}{1+\alpha}} \left(\frac{1+\tau}{\lambda}\right)^{\frac{2\alpha}{1+\alpha}}$$

The condition of intersections between the SS curve and the WW curve can be translated into the condition of the following inequality

$$
\left\{ (m^S)^{\frac{1-\alpha}{1+\alpha}} \beta L^S - \frac{2(1+\alpha)}{1-\alpha} (m^S)^{\frac{2}{1+\alpha}} f^S_m \right\}_{\text{max}} > (1-\beta)(2\mu^S)^{\frac{1-\alpha}{1+\alpha}} \left(\frac{1+\tau}{\lambda}\right)^{\frac{2\alpha}{1+\alpha}}
$$

The L.H.S. of the above inequality obtains its maximum value when

$$m^S = \frac{(1-\alpha)^2 \beta L^S}{4(1+\alpha)f^S_m}, \text{ put this exact value into the above inequality, we can obtain inequality (24).}$$

**Condition 2:** both of the points of intersection lies above the horizontal line $\omega=1$, which can be represented by the following inequality

$$\frac{2(1-\beta)}{1+\alpha} L^N < \beta L^S < (1-\beta)L^N + \frac{4(1+\alpha)\mu^S}{1-\alpha} \left(f^S_m\right)^{\frac{1-\alpha}{1+\alpha}} \left(\frac{1+\tau}{\lambda}\right)^{\frac{2\alpha}{1+\alpha}}$$

(25)

Proof: from equation (21), we get $m^S = 2\mu^S \left(\frac{1}{\omega}\right)^{\frac{1-\alpha}{1+\alpha}} \left(\frac{1+\tau}{\lambda}\right)^{\frac{2\alpha}{1+\alpha}}$, combine this expression with equation (19), we have the following equation

$$\omega^{\frac{1-\alpha}{1+\alpha}} \left[\beta L^S - (1-\beta)\omega L^N\right] = \frac{4(1+\alpha)\mu^S f^S_m}{1-\alpha} \left(\frac{1+\tau}{\lambda}\right)^{\frac{2\alpha}{1+\alpha}}$$

(26)

We can illustrate the above equation in the following figure
The curve in Figure depicted the L.H.S. of (26), while the horizontally dotted line depicted the R.H.S. of (26), thus the intersection point of this horizontally dotted line with the vertical axis is point \(0, \frac{4(1+\alpha)\mu^s f_m^s}{1-\alpha} \left(\frac{1+\tau}{\lambda}\right)^{\frac{2\alpha}{1-\alpha}}\). If the abscissas of the two points of intersection of the curve and horizontally dotted line illustrated in Figure 3 are both bigger than 1, we will get the exact conclusion of our condition 2.

Put \(\omega = 1\) into the L.H.S. of (26), we get the first of the related conditions, which can be expressed by the following inequality

\[
\beta L^s - (1-\beta)L^\bar{y} < \frac{4(1+\alpha)\mu^s f_m^s}{1-\alpha} \left(\frac{1+\tau}{\lambda}\right)^{\frac{2\alpha}{1-\alpha}}
\]

The second related condition has the intuition that the slope of the curve representing the L.H.S. of (26) is positive at the exact point whose abscissa is \(\omega = 1\). This condition can be expressed by the following inequality

\[
\beta L^s > \frac{2(1-\beta)}{1+\alpha} L^\bar{y}
\]

Thus we can get the two inequalities concerning the original condition, which have already been illustrated in Condition 2.

### 4. Comparative statics

In this section, we study the effect of several factors, including tariff level, the technologies for customization, the sizes of the two countries and so on, on the pattern of organism of final-good producers in the north. We begin with tariff level, because this is the focus of our paper.

#### 4.1. Tariff level

Consider the decrease in tariffs levied upon the components from south to north,
which can be reflected as a smaller value of \( \tau \). An initial stable equilibrium with outsourcing in the south can be illustrated by point \( E \) in the following figure.

A decrease in tariff level causes the WW curve to move downward, because for given \( m^S \), it will be more profitable for final-good producers to choose the mode of outsourcing at the stage of entry and search for partners in the south at the initial relative wage. Thus, the relative wage must decrease to keep equal profitability between the mode of outsourcing from and vertical integration. Therefore, the SS curve shifts down, which causes the equilibrium to move from \( E \) to \( E' \). We can see that at the new equilibrium, the total number of component suppliers in the south is larger and the relative wage is smaller.

![Figure 4: reduction of tariff level](image)

Now we examine the relative numbers of both those final producers choosing the mode of outsourcing and those choosing the mode of vertical integration in the north. If we combine equations (10), (11), (12) and (17) together, we can get the following equation expressing the total number of final-good producers searching partners in the south

\[
2 \frac{m^S f^S}{f_v \omega} = n^S_2
\]

It shows that the total number of final producers choosing the mode of outsourcing from abroad in the north has a positive relationship with the total number of component suppliers in the south and a negative relationship with the relative wage rate. The former relationship reflects a positive feedback mechanism, with more component producers, a final producers choosing the mode of outsourcing is more likely to find a partner willing to undertake the exact investment in customization, so there may be more such kind of final producers. When there are more final producers choosing the mode of outsourcing, it may be more profitable for a component supplier to enter the market.
On the other hand, from equation (20), we have the following equation concerning the total number of final suppliers choosing the mode of vertical integration and some factors, including the relative wage and the total number of component suppliers

\[ n_i = \frac{(1-\alpha)L^N}{f_v} - \frac{2(1-\alpha)m^S f^S}{f_v \omega} \]  

(28)

It is easy to find that the number of final producers choosing the mode of vertical integration will increase if the relative wage increases, and decrease if the total number of component suppliers in the south increases, which has a similar interpretation as our analysis of the number of the other kind of final producers.

Then what’s the effect on the volume of outsourcing in the south of a decrease in the tariff level? We know that the volume of outsourcing is exactly \( v^S = 2m^S r^S n_3 y^S \), which is the total number of units of components manufactured by input suppliers in the south. Together with (5), (6), (12) and (13), we have the following equation

\[ v^S = \frac{4\alpha m^S f^S}{(1-\alpha)(1+\tau)} \]  

(29)

Thus it is easy to say that with the decreasing of tariffs, which may cause the total number of component suppliers to increase, the volume of outsourcing activity will rise.

4.2. Country size

Assume that there is a certain growth in the resource endowment of the south, which can be reflected in an increased in the total labor supply \( L^S \), we can illustrate the relevant changes by the following figure.

The initial equilibrium is at point \( E \), and the growth of labor supply in the south shifts the SS curve upward, the reason is that for given number of component suppliers, the added labor exceeds the relevant needed in the production of homogeneous goods, and this will cause the relative wage in the south to decrease. The new SS curve is represented by the broken curve in the above figure. Since there is no movement of the WW curve, the new equilibrium is now at point \( E' \). We can see that the number of component producers in the south turns out to be larger and the relative wage becomes lower.
Figure 5: increase in labor supply in the south

Then how to comprehend the fact that the relative wage will decrease implied by the comparison between the initial equilibrium and the new equilibrium, the direct effect of an increase in the labor supply in the south is that it will cause the relative wage to increase. But the lower wage rate makes final producers to find that it is more profitable to choose the mode of outsourcing from abroad, with more final producers searching in the south, more component producers may enter the market because of profits chance. And the shift to outsourcing activity has an opposite effect, moreover, the thick-market externality means that outsourcing is an increasing activity. To keep the equal profitability between choosing the mode of outsourcing from abroad and vertical integration, the relative wage at the new equilibrium must be lower than that at the initial equilibrium due to the thicker market in the south now.\(^6\)

Once we know that the number of component producers is larger and the relative wage is lower, we can conclude the fact that there are more final producers choosing the mode of outsourcing from abroad from equation (27), and that the total number of final producers choosing the mode of vertical integration in the north decreases from equation (28). When it comes to the volume of outsourcing, we can judge that this data becomes larger from equation (29). The relevant method of analysis is just similar to what we have used in the analysis of the effect of a small decrease in tariffs on those variables.

We will not repeat the case of an increase in the north’s labor supply, which has opposite effects on relative wage, the total number of component suppliers in the south, the total number of final producers choosing the mode of outsourcing and that

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\(^6\) That the rise in the supply of an input may lead to a rise in its relative reward has been discussed in other situations as well. For example, Grossman and Helpman (1991) find that in a world in which the north innovates and the south imitates an increase in the size of the south may raise its wage level.
of final producers choosing the mode of vertical integration to the effects of an increase in the south’s labor supply.

4.3. Outsourcing technology

The technology for outsourcing from abroad can be reflected by the variable $\mu^S$ that represents the cost of customizing a prototype for a particular producer of final goods. It seems that with the development of technology, the cost of customizing a prototype is decreasing, which can be represented by a smaller value of $\mu^S$. The exact effect of the development of outsourcing technology can be illustrated by the following figure.

![Figure 6: improvement in outsourcing technology](image)

When $\mu^S$ falls, it will be more profitable for final producers to choose the mode of outsourcing from abroad and search for partners in the south given the initial relative wage unchanged. To restore equal profitability, the relative wage rate in the north must decrease, which may causes the WW curve to move downward, as illustrated in Figure 6. We can see that the equilibrium moves form point $E$ to point $E'$, which implies an increase in the number of component suppliers in the south and a decrease in the relative wage.

Next we turn to the pattern of organization of final producers in the north, with $m^S$ increasing and $\omega$ decreasing, we can get that the total number of final-good producers choosing the mode of outsourcing from abroad increases according to equation (27), and the number of those choosing the mode of vertical integration decreases from equation (28). When it comes to the volume of outsourcing between the south and north, the exact volume will increase based on equation (29). Thus we end the analysis of effects of improvement in outsourcing technology on the relevant
Next, we examine the effects of improvements of input-output coefficient of vertically integrated producers, which can be represented by the parameter $\lambda$. We know that $\lambda$ measures the exact units of northern labor needed to produce one unit of components by those vertically integrated producers in north, therefore, a fall in $\lambda$ may reflect an improvement in management or an improvement in specialization within vertically integrated firms. Based on equations (21) and (19), we can draw the following graph, which illustrates the relevant changes.

As the input-output coefficient decreases, it may be more profitability for final-good producers to choose the mode of vertical integration at the stage of entry given the total number of component suppliers in the south unchanged. To restore equal profitability, the relative wage must rise in the north, thus the WW curve moves upward, we can see from Figure 7 that such a shift will cause the initial equilibrium to move from point $E$ to point $E'$, which indicates a higher relative wage and a smaller number of component suppliers in the south.

The results of an improvement in the input-output coefficient of vertically integrated producers are as follows: the number of final-good producers choosing the mode of outsourcing from abroad decreases and the number of those choosing the mode of vertical integration increases, which can be concluded from (27) and (28). We may anticipate that such a kind of improvement can lessen the total volume of outsourcing since it brings about relatively more advantages to vertically integrated firms, and out result from equation (32) confirms this anticipation, with an increase in
relative wage and a fall in the quantity of component suppliers.

Therefore, the increase in outsourcing in the global economy recently may have its own reasons, from the above analyses, the decreasing tariff level, the increasing labor supply of those developing countries and the improvement in customization due to technology development may all help to boost the race of outsourcing in the global economy.

5. Conclusions
We have developed a model based on the work of Grossman and Helpman (2002) to analyze the determinants of the exact pattern of organization in industry equilibrium. In our model, final producers of differentiated goods can choose the mode of its organization, either outsourcing from abroad or vertical integration. For those choosing vertical integration, they must face the high costs of production of components themselves, perhaps because of lack of specialization and management hardship, but they don’t have to pay money for the trade of component and search costs in the south. For those choosing outsourcing from abroad, they must face the costs of tariffs levied on the trade of components and the costs of search for partners in the south, but the costs of production of components by those input suppliers in the south is lower than that of vertical integration, when those final producers share profits with component suppliers, this is an important advantage for them. What we try to do is to found an equilibrium between those two kinds of modes for final producers, and to examine the effects of some economic variables, such as tariffs level, labor supply, improvement in management for vertically integrated firms and technology improvement in customization on the exact pattern of organization for final-good producers.

Our model assumes a thick-market externality: for final producers searching in the south for their partners, the more component suppliers there are, the more profitable it is; for component suppliers, the more final producers searching for partners, the better they run. The externality is why we may have multiple equilibria, while our attention is focused on the stable equilibrium in which some final producers choose the mode of vertical integration while others choose the mode of outsourcing from abroad.

A fall in tariffs level may cause an increase of the number of those final producers choosing outsourcing from abroad and a decrease of the number of final producers choosing the mode of vertical integration, what’s more, the total volume of outsourcing in the global economy increases. An increase in the south’s labor supply and an improvement in customization have a similar effect as a fall in tariffs, both will
encourage the activity of outsourcing from abroad, and will cause the pattern of organization for final producers to shift from vertical integration to outsourcing from abroad, with the volume of outsourcing increasing together. If we turn to the effects of an improvement in management and specialization for those vertically integrated producers, or an increase in the north’s labor supply, the effects are exactly opposite. The pattern of organization for final producers will shift from outsourcing to vertical integration, which implies that the number of final producers choosing the mode of outsourcing from abroad decreases and the number of the other kind of final producers will increase.

We do not consider much about the contracting environment, but simply assume that there isn’t any order contract between final producers searching partners in the south and component suppliers. Due to the fact that the improvement in contracting environment may also affect the pattern of organization for final producers, we can examine this effect in future as extensions.

REFERENCES