

# Investment in Human Capital Accumulation and Growth: Analyzing of Policy Effectiveness

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## Introduction

The endogenous growth framework has become a useful tool to evaluate the long-run growth consequences of public policy since the pivotal work by Romer (1986) and Lucas (1988). There are substantial informational and institutional barriers to labor search, recruiting, and job creation. This paper integrates an endogenous growth model into a labor search model *a la* Mortensen (1982) and Pissaridis (1984) and uses the model to evaluate the effectiveness of some forms of human capital-related policies. We emphasize that labor market frictions can have long-run growth and welfare implications.

We build a two-sector endogenous growth model with physical and human capital accumulation wherein the labor market is subject to search and entry frictions. Both vacancy creation and job search are costly and vacancies and job seekers are brought together by a constant-returns matching technology. A Mortensen (1982)-Pissaridis (1984) based model is difficult to calibrate to match fundamental observations in the labor market

<sup>1</sup> We consider large firms and large households in the sense that each firm can create multiple vacancies and each household can choose labor-market participation endogenously. These features allow us to move one step closer to the canonical endogenous growth setup for quantitatively conducting policy analysis.

We calibrate the model to match the U.S. economy and then perform

comparative-static analysis and policy evaluation. The main findings are as follows. First, employment, labor-market participation, job creation, learning effort and output growth rise with an increase in the effectiveness of human capital accumulation (i.e., the degree of labor-market matching efficacy), but decrease with the job separation rate and the job creation cost. Next, a shift in these parameters fostering long-run growth is always accompanied by a higher unemployment rate because, in response to such shifts in growth-enhancing parameters, the labor market also becomes tighter from the firm's view point. Finally, an enhancement in human capital that influences learning is more effective in fostering growth and it is also associated with a larger decline in effective consumption and leisure as well as a larger increase in the unemployment rate.

We carry out quantitative policy evaluation of two human capital policy programs. One program does not directly affect households' learning effort and the other does. The former human capital enhancement policy is more of an experience enhancement policy, and the latter is more of an on-the-job training policy.<sup>2</sup> Under a constant government budget, an on-the-job training policy is more effective in promoting labor-market participation, learning, employment and economic growth than a human capital enhancement policy. In spite of its strong positive growth effect, however, an on-the-job training policy also leads to a larger drop in effective consumption and aggregate leisure for the employed, thereby reducing welfare despite its strong positive growth effect. When the labor-market frictions are less severe, the effects of these human capital policy programs become smaller. This suggests that a quantitative evaluation of the effectiveness of labor-related policy in a Walrasian world is downward biased.

The organization of the paper is as follows. In the next section, we set up the model. Optimization is studied in the third section and the equilibrium in the fourth section. In the fifth section, we carry out the numerical analysis. Finally, concluding remarks are offered in the final section.

## **The Model**

The model has a continuum of identical infinitely lived competitive firms, a continuum of identical infinitely lived households and the government. All agents have perfect foresight. There are two productive factors: capital and labor, both owned by households. Firms and households exchange in both goods and factor markets. The goods market and the capital market are perfect, but the labor market exhibits search and entry frictions. Each firm can create multiple vacancies and each household can choose search intensity endogenously. Vacancy creation and search intensity are costly.

We adopt the large household framework. Each household is thought of containing a continuum of members with their mass normalized to unity. All members pool their income as well as their consumption and leisure. Employed members engage in production, learning, or leisure activities. Non-employed members engage in job seeking or leisure activities. This structure avoids unnecessary complexity involved in managing the distribution of the employed, the unemployed and their respective human and non-human wealth. Vacancies and job seekers are brought together through a matching technology with one vacancy filled by exactly one searching worker. Filled vacancies and workers are separated every period at an exogenous rate and separated workers immediately become job seekers. Finally, the government determines tax rates and human capital enhancement

policies that balance the budget in each period.

### Firms

In each period, the representative large firm rents capital  $k_t$  and employs  $n_t$  labor with effort  $l_t$  to produce a single final good  $y_t$ . Moreover, to post and maintain the vacancies  $v_t$ , a mass of workers of measure  $\Phi$  are employed to the vacancy creation cost of posting vacancies, managing personnel-related documentations, as well as maintaining the office space.

We postulate  $\Phi(v_t) = \phi v_t^\varepsilon$ , where  $\varepsilon > 0$  reflects the convexity of the vacancy creation cost and  $\phi > 0$  captures any exogenous shift in such a cost. Accordingly, workers used for manufacturing are  $(n_t - \Phi(v_t))$ . The output of the representative firm is

$$y_t = A k_t^\alpha \left[ (n_t - \Phi(v_t)) l_t h_t \right]^{1-\alpha} \quad (1)$$

where  $l_t$  is effort and  $h_t$  is human capital of workers,  $\alpha \in (0,1)$  is the output elasticity of capital and  $A > 0$  denotes the scaling factor of the production technology.

The rate of return on capital is  $r_{k_t} = A\alpha \left[ \frac{k_t}{(n_t - \Phi(v_t)) l_t h_t} \right]^{\alpha-1}$ , and is a decreasing function of the effective capital-labor ratio, denoted by  $q_t$ . We can rewrite this expression as

$$q_t \equiv \frac{k_t}{(n_t - \Phi(v_t)) l_t h_t} = \left( \frac{A\alpha}{r_{k_t}} \right)^{\frac{1}{1-\alpha}} \quad (2)$$

### Households

Facing a pooled resource, a representative large household has a unified preference for all its members: the employed  $n_t$  and the non-employed  $1-n_t$ . To simplify the analysis, we restrict our attention primarily to the case wherein only the employed devote time to

accumulating human capital, thus only on-the-job learning. Employed members divide their time into production  $l_t$  (work effort), human capital investment  $e_t$  (learning effort) and leisure  $1-l_t-e_t$ . Non-employed members divide their time only into job search  $s_t$  (search effort or search intensity) and leisure  $1-s_t$ . The search intensity augmented unemployment measure is defined as  $u_t=s_t(1-n_t)$ .

In addition to leisure, members of the representative household also value their pooled consumption  $c_t$ . The representative household's periodic felicity function is given by,

$$U(c_t, l_t, e_t, s_t, n_t) = u(c_t) + n_t \Lambda^1 (1 - l_t - e_t) + (1 - n_t) \Lambda^2 (1 - s_t) \quad (3)$$

where employed and non-employed members need not value their leisure time equally ( $\Lambda^1$  and  $\Lambda^2$  may differ). Functions  $u$  and  $\Lambda^1$  and  $\Lambda^2$  are strictly increasing and concave. Thus, the representative household's preference can be written in a standard time-additive form as:

$$\Omega = \sum_{t=0}^{\infty} \left( \frac{1}{1+\rho} \right)^t U(c_t, l_t, e_t, s_t, n_t) \quad (4)$$

where  $\Omega$  is the lifetime utility and  $\rho > 0$  is the subjective rate of time preference.

Finally, we extend Lucas (1988) to specify human capital evolution as:

$$h_{t+1} = (1 + \zeta + D n_t e_t) h_t, \quad \zeta > 0, \quad D > 0, \quad h_0 > 0, \quad (5)$$

where  $\zeta$  is the exogenous rate at which human capital is accumulated,  $D$  is the maximum rate of endogenous human capital accumulation, and  $h_0$  is initial human capital prior to entry into the labor market after completing mandatory formal schooling.

In our policy analysis, we shall refer to an increase in  $\zeta$  as an experience enhancement policy and that to raise  $D$  as an on-the-job training policy. As argued by Heckman (1976), better formal education not only increases the higher *level* of initial human capital prior to entering the labor market ( $h_0$ ) but also raises the *rate* at which human capital is accumulated. One may thus regard mandatory education (K-12) as to increase  $\zeta$  and college education as to increase  $D$ . These policy programs are expected to affect human capital accumulation differently.

## Optimization

### The Aggregate Economy

Because there is only a single good in the economy, the resource constraint requires that aggregate goods supply must be equal to aggregate goods demand, which is the sum of households' consumption and gross investment:

$$c_t + [k_{t+1} - (1 - \delta)k_t] = Ak_t^\alpha [(n_t - \Phi(v_t))l_t h_t]^{1-\alpha}, \quad \delta > 0, \quad (6)$$

where  $\delta \in (0, 1)$  is the depreciation rate of capital.

The labor market exhibits search frictions. According to Diamond (1982), the matching technology exhibits constant returns. The matching technology is

$$m_t = B [s_t (1 - n_t)]^\beta v_t^{1-\beta}, \quad B > 0, \quad \beta \in (0, 1) \quad (7)$$

where  $\beta \in (0, 1)$  measures the contribution of a job seeker in a match and  $B > 0$  measures the degree of matching efficacy.

Employment within the economy thus evolves according to the following birth-death

process:  $n_{t+1}-n_t = m_t - \psi n_t$ , where  $\psi$  is the (exogenous) job separation rate. By using (7), employment evolves according to

$$n_{t+1} = (1 - \psi)n_t + B[s_t(1 - n_t)]^\beta v_t^{1-\beta} \quad (8)$$

Thus,  $m_t/v_t \equiv \eta_t$  is the firm recruitment rate and  $m_t/[s_t(1 - n_t)] \equiv \mu_t$  is the job finding rate.

Following the arguments in Merz (1995) and Andolfatto (1996), we assume that a decentralized economy will have the same outcome as the pseudo social planner's problem. It can be shown that this requires the supporting wage to have the households' bargaining share equal to the corresponding matching elasticity,  $\beta$ ; that is, Hosios' (1990) rule holds. With this equivalence property, we can focus on the pseudo social planner's problem. In particular, the policy evaluation herein is on contrasting two human capital policy programs, one that favors those devoted more time to learning and another that treats everyone identically. Since both policies only affect human capital accumulation without affecting households' budget constraints or firms' flow profits, it is valid to carry out equilibrium and welfare analysis based exclusively on the pseudo social planner's problem. Note that the optimization problem is a pseudo social planner's problem in the sense that the social planner cannot fully coordinate search/matching and that the social planner takes prices as given when considering policy programs.

## Optimization

The dynamic programming problem is specified as the following Bellman equation

$$\Omega(k_t, h_t, n_t) = \max_{c_t, l_t, e_t, s_t, v_t} U(c_t, l_t, e_t, s_t, n_t) + \frac{1}{1 + \rho} \Omega(k_{t+1}, h_{t+1}, n_{t+1}) \quad (9)$$

subject to constraints (5), (6), and (8).

It is easy to derive the first-order conditions with respect to consumption ( $c$ ), work effort ( $l$ ), learning ( $e$ ), and search intensity ( $s$ ) and vacancy creation ( $v$ ), as well as the Benveniste-Scheinkman conditions governing the two capital stocks and the level of employment ( $k, h, n$ ). We will use prime to indicate values in the next period. We can easily manipulate the first-order conditions to obtain the following intratemporal and intertemporal trade-off relationships:

$$-\frac{U_l}{U_c} = (1 - \alpha)Aq^\alpha(n - \Phi)h \quad (10a)$$

$$\frac{W_h(k', h', n')}{(1 + \rho)} \cdot (Dnh) = -U_e \quad (10b)$$

$$\frac{W_n(k', h', n')}{(1 + \rho)} \cdot [\beta\mu(1 - n)] = -U_s \quad (10c)$$

$$\frac{W_n(k', h', n')}{(1 + \rho)} \cdot [(1 - \beta)\eta] = U_c \cdot [(1 - \alpha)Aq^\alpha lh\Phi_v(v)] \quad (10d)$$

Condition (10a) displays a standard consumption-leisure trade-off by equating the marginal rate of substitution with the marginal product of labor. For the other relationships, denote that  $MVH' = \Omega_h(k', h', n')/(1 + \rho)$  is the marginal valuation of additional human capital accumulated in the next period is and  $MVN' = \Omega_n(k', h', n')/(1 + \rho)$  is the marginal valuation of additional employment to be used in production in next period, where subscripts denote derivatives. Noting that  $Dnh$  measures incremental human capital accumulated as a result of learning, condition (10b) requires that the future net gain from learning, by enhancing human capital and hence productivity, be equal to the current loss



from a reduction in leisure. With  $\beta\mu(1-n)$  representing the incremental employment as a consequence of more effort devoted to finding a job, (10c) states that the employment gain next period from a marginal increase in search intensity this period equals the disutility from the corresponding reduction in leisure. Finally, with  $(1-\beta)\eta$  representing the incremental employment as a consequence of more vacancy created to recruit workers, (10d) indicates that the marginal benefit of vacancy as a result of a successful recruitment equals the sacrifice in the labor used for production in order to maintain the additional vacancy created, where  $(1-\alpha)Aq^\alpha h\Phi_v(v)$  is the marginal cost of vacancy in units of goods due to a loss of labor productivity.

Moreover, we can combine the first-order conditions and the Benveniste-Scheinkman condition to obtain the following intertemporal trade-off relationships:

$$(1 + \rho) \frac{U_c}{U_{c'}} = (1 - \delta) + \alpha A(q')^{\alpha-1} \quad (11a)$$

$$MVH \cdot h = -U_l l + \left(1 + \frac{1 + \zeta}{Dne}\right) (-U_e e) \quad (11b)$$

$$MVN \cdot n = U_n n - U_e e + \frac{n}{n - \Phi} (-U_l l) + \frac{n}{1 - n} \frac{1 - \psi - \beta\mu s}{\beta\mu s} (-U_s s) \quad (11c)$$

While (11a) is a standard intertemporal consumption-saving trade-off condition, equating the marginal rate of intertemporal substitution with the rate of returns on capital, (11b) governs the evolution of human capital. Condition (11c) dictates the evolution of employment. These relationships equate next period's marginal valuation of incremental human capital and incremental employment, respectively, with the corresponding net marginal opportunity cost from the productivity loss today. It should be noted that, if the

employed value leisure more than the non-employed, the marginal opportunity cost of incremental employment is dampened by an increase in the marginal utility of leisure resulting from having more employed members in the large household (measured by  $U_n n$ ).

## Equilibrium

A *dynamic search equilibrium* is a tuple of individual choice variables,  $\{c_t, l_t, w_t, s_t, v_t, y_t\}_{t=0}^{\infty}$ , state variables,  $\{k_{t+1}, h_{t+1}, n_{t+1}\}_{t=0}^{\infty}$ , and aggregate variables,  $\{m_t, r_{kt}, q_t\}_{t=0}^{\infty}$ , such that:

- (i) all individuals optimize, i.e., (10a)-(10d) and (11a)-(11c) are met;
- (ii) human capital and employment evolve according to (5) and (8), respectively;
- (iii) goods production is given by (1) and the effective capital-labor ratio satisfies (2);
- (iv) labor-market matching satisfies (7);
- (v) the goods market clears, i.e., (6) holds.

The model economy exhibits perpetual growth. To analyze the economic aggregates, we need to transform perpetually growing quantities into stationary ratios. We focus on a *balanced growth path* (BGP) along which consumption, physical and human capital, and output all grow at positive constant rates. Since the production function is homogeneous of degree one in  $k$  and  $h$  and the human capital accumulation equation is linear in  $h$ , these quantities ( $c$ ,  $k$ ,  $h$  and  $y$ ) must all grow at a common rate,  $g$ , on a BGP, whereas other quantities are all constant.

Along a BGP, the labor market must satisfy the steady-state matching relationships (Beveridge curve) given by,

$$m = \psi n = \mu s(1 - n) = \eta v = B[s(1 - n)]^\beta v^{1-\beta} \quad (12)$$

That is, the equilibrium outflows from the matched pool ( $\psi n$ ) must equal the inflows from either the unmatched worker pool ( $\mu s(1-n)$ ) or the unmatched job vacancy pool ( $\eta v$ ).

For analytical convenience, we assume the felicity function to take the following form:

$$u(c)=\ln c, \Lambda^1(1-l-e)=\gamma_1(1-l-e)^{1-\sigma}/(1-\sigma) \text{ and } \Lambda^2(1-s)=\gamma_2(1-s)^{1-\sigma}/(1-\sigma),$$

where  $\gamma_i > 0$  and  $\sigma > 0$ .

While a log utility in consumption ensures bounded lifetime utility, employed and non-employed members value leisure differently only by scaling factors  $\gamma_1$  and  $\gamma_2$ . It is convenient to write the ratio of the marginal utility of leisure of employed to unemployed members as  $R=\gamma_1(1-l-e)^{-\sigma}/[\gamma_2(1-s)^{-\sigma}]$ . Hence the marginal utility of additional employment can be calculated as

$$U_n=\Lambda^1-\Lambda^2=\gamma_2(1-s)^{-\sigma}[(1-l-e)R-(1-s)]/(1-\sigma),$$

which is expected to be positive in our benchmark economy.

Along a BGP, first, we can rewrite the two evolution equations (5) and (6), as:

$$e = \frac{g - \zeta}{Dn} \quad (13a)$$

$$\frac{c}{h} = [Aq^\alpha - (\delta + g)q](n - \Phi)l \quad (13b)$$

Next, we can show that

$$g = \frac{r_k - (\delta + \rho)}{1 + \rho} \quad (14a)$$

$$\rho(1 + g) = Dnl \quad (14b)$$

$$\frac{\rho + \psi}{\beta\mu} + \frac{1 - \sigma s}{1 - \sigma} = R \left( \frac{nl}{n - \Phi} + \frac{1 - l - \sigma e}{1 - \sigma} \right) \quad (14c)$$

$$\frac{\Phi_v n l R}{n - \Phi} = \frac{(1 - \beta)\eta}{\beta\mu} \quad (14d)$$

While (14a) gives the prototypical Keynes-Ramsey relationship that governs consumption growth, (14b) is a relationship based upon intertemporal human capital accumulation. Condition (14c) is one based on intertemporal employment evolution and (14d) is one based on the vacancy creation trade-off.

Using (14a) and (2), we have:

$$r_k = (\delta + \rho) + (1 + \rho)g \quad (15a)$$

$$q = \left[ \frac{A\alpha}{(\delta + \rho) + (1 + \rho)g} \right]^{\frac{1}{1-\alpha}} \quad (15b)$$

Both relationships are standard in discrete-time optimal growth models with a Cobb-Douglas production technology. We can substitute out  $c/h$  and  $q$  in (13b) to yield:

$$\frac{\rho + \psi}{\beta\mu} + \frac{1 - \sigma s}{1 - \sigma} = R \left( \frac{nl}{n - \Phi} + \frac{1 - l - \sigma e}{1 - \sigma} \right) \quad (16)$$

where the right-hand side is increasing in  $g$  and the left-hand side may also be locally increasing in  $g$ . Note that the fixed point mapping may lead to multiple solutions for  $g$ . In practice, it is easier to solve the problem if the system is reduced to two dimensions. Below, we will reduce the system to two dimensions.

### The System of Two-by-Two

The equations determining the BGP can be re-arranged in a recursive fashion that is conducive to perform comparative statics. Essentially, we can reduce the system to  $2 \times 2$  in  $(\mu, n)$  space. Once the BGP values of  $(\mu, n)$  are pinned down, the rest of endogenous variables can then be derived recursively.

To see this, we use (12) to derive:

$$\eta = B^{\frac{1}{1-\beta}} \mu^{\frac{-\beta}{1-\beta}} = \eta(\underbrace{\mu}_{(-)}; \underbrace{B}_{(+)}) \quad (17a)$$

$$v = B^{\frac{-1}{1-\beta}} \mu^{\frac{\beta}{1-\beta}} \psi n = v(\underbrace{\mu}_{(+)}, \underbrace{n}_{(+)}; \underbrace{B}_{(-)}, \underbrace{\psi}_{(+)}) \quad (17b)$$

$$s = \frac{\psi n}{(1-n)\mu} = s(\underbrace{\mu}_{(-)}, \underbrace{n}_{(+)}, \underbrace{\psi}_{(+)}) \quad (17c)$$

The properties regarding (17a) are standard: while an increase in  $B$  represents an outward shift in the Beverage Curve that tends to raise both job finding rate and firm recruitment rate, any other parameter changes cause a movement along the Beverage Curve in  $(\mu, \eta)$  space and hence affect the job finding rate and firm recruitment rate differently. Accordingly, an increase in  $B$  fosters more matches and hence reduces unfilled vacancies; however, an increase in the job finding rate is associated with a reduction in the firm recruitment rate, leading to more unfilled vacancies. Additionally, a higher job separation rate raises unfilled vacancies whereas an increase in employment requires creation of more vacancies to match. The last relationship is a direct consequence of the first equality in (12): a higher job finding rate enables workers to devote less effort to job search and a higher job separation rate requires workers to spend more search effort.

Then, from (14b) and (13a), we can write learning effort  $e$  as:

$$e = \frac{l}{\rho} - \frac{1+\zeta}{Dn} \quad (18)$$

which is positively related to both employment and work effort.

It is straightforward to pin down work effort as:

$$l \left( 1 + \frac{1+\zeta}{Dn} - \frac{1+\rho}{\rho} l \right)^{-\sigma} = \frac{(1-\beta)\eta}{\beta\mu} \frac{n-\Phi}{n\Phi_v} \frac{\gamma_2(1-s)^{-\sigma}}{\gamma_1} \quad (19a)$$

which can be rewritten as an implicit function:

$$l = l(\underset{(-)}{\mu}, \underset{(?) }{n}; \underset{(-)}{B}, \underset{(?) }{\psi}, \underset{(-)}{\phi}, \underset{(+)}{D}, \underset{(-)}{\zeta}) \quad (19b)$$

That is, work effort can be expressed as a function of  $(\mu, n)$  alone. A higher job finding rate fosters more matches and, as a result of diminishing returns, lowers the marginal benefit of additional employment (measured by  $\Omega_n(H')$ ). In our production function, employment and work effort are Pareto complements, so the marginal benefit of work effort decreases. This explains why work effort is negatively related to the job finding rate. An increase in employment creates two opposing effects. It, on the one hand, lowers the marginal benefit of employment (by diminishing returns) and hence the marginal benefit of work effort. On the other, it increases the marginal benefit of work effort as a result of Pareto complementarity. On balance, we have an ambiguous relationship between work effort and employment. Since the effects of exogenous parameters are all partial effects for given values of  $(\mu, n)$ , we will not devote our time to discussing the details but will return to these issues in the numerical analysis after solving each of the endogenous variables in terms of exogenous parameters.

Using (19b), we write (14b), (15a) and (15b), respectively, as:

$$g = g(\underset{(-)}{\mu}, \underset{(?) }{n}; \underset{(+)}{B}, \underset{(?) }{\psi}, \underset{(-)}{\phi}, \underset{(+)}{D}, \underset{(-)}{\zeta}) \quad (20a)$$

$$r_k = r_k(\underset{(-)}{\mu}, \underset{(?) }{n}; \underset{(+)}{B}, \underset{(?) }{\psi}, \underset{(-)}{\phi}, \underset{(+)}{D}, \underset{(-)}{\zeta}) \quad (20b)$$

$$q = q(\underset{(-)}{\mu}, \underset{(?) }{n}; \underset{(+)}{B}, \underset{(?) }{\psi}, \underset{(-)}{\phi}, \underset{(+)}{D}, \underset{(-)}{\zeta}) \quad (20c)$$

Since the balanced growth rate is positively related to work effort and work effort is negatively related to the job finding rate, we immediately establish the relationship between the balanced growth rate and the job finding rate for a given level employment. The ambiguity between work effort and employment is also carried over, leading to an ambiguous relationship between balanced economic growth and employment.

Finally, if we substitute (19b) and (20a)-(20c) into (14c) and (16), we obtain two fundamental relationships that jointly pin down  $(\mu, n)$ . The relationship derived from (14c) is referred to as the pseudo labor supply locus (LS) and the relationship obtained from (16) can be called the pseudo labor demand locus (LD). Intuitively, the LS locus represents how labor supply responds to a better labor market condition as a result of a higher job finding rate (higher  $\mu$ ), whereas the LD locus indicates how labor demand changes in response to a tighter labor market from the viewpoint of employers (higher  $\mu$  or lower  $\eta$ ). These schedules are pseudo demand and supply because both schedules are in terms of a job matching probability  $\mu$  in lieu of wages and because both relationships have incorporated goods market clearance and labor matching equilibrium conditions. While the direct effects are to yield an upward-sloping LS locus and a downward-sloping LD locus, there are several indirect effects present in our dynamic general equilibrium models making the net effects ambiguous. The ambiguity of the underlying indirect effects include the potential conflicts between (i) the substitution and the wealth effects, (ii) the employed and the non-employed within each household, and (iii) households and firms. Of course, the elastic work effort and learning effort as well as the variable vacancies created by each firm lead to further complexity and ambiguity. Nonetheless, one assumes log-linear utility to

remove the first potentially conflicting forces and restricts the non-employed to have less marginal enjoyment in leisure to remove the second ambiguity. If some forms of normality in matching and in labor allocation are further imposed, one may then expect an upward-sloping LS locus in conjunction with a downward-sloping LD locus.

Due to the aforementioned complication in general, we will not perform any further analytic characterization, but instead defer the comparative static analysis to the next section using a numerical method by calibrating the model based on the U.S. data.<sup>3</sup> As will be illustrated, our calibrations will reconfirm the benchmark case with well-behaved upward-sloping LS locus and downward-sloping LD locus.

We should mention that if there is no search and match friction, capital rental ( $r$ ) and wage rate ( $w$ ), take the following forms.

$$1 + r = 1 + r_k = (1 + \rho) \frac{U_c}{U_c} \quad (21a)$$

$$w = \left[ \beta + (1 - \beta) \left( 1 - \frac{\Gamma}{1 - \beta} \right) \right] \bar{w} > \bar{w} \equiv \frac{n - \Phi}{n} MPL \quad (21b)$$

Thus, the supporting wage with frictional labor markets is lower than the competitive wage.

The wage discount is:  $\Gamma = \frac{1 - \beta}{\beta} \frac{1 + \rho}{R\ell\mu} \frac{r_k + \psi - g(1 - \psi)}{1 + r_k} > 0$ .

## Numerical Analysis

We now turn to quantifying our results by calibration analysis. Moreover, we provide a policy analysis by assessing the growth effects and the welfare consequences of an array



of labor-market related subsidies.

## Calibration

We calibrate parameter values to match the U.S. quarterly data over the period of 1951-2003. See data of the observables and benchmark parameter values in Table 1. The quarterly per capita real GDP growth rate is set to  $g=0.45\%$  and the quarterly depreciation rate of capital is set to  $2\%$  to match the annual per capita real GDP growth rate of  $1.8\%$  and the annual depreciation rate of capital in the range of  $5-10\%$ , respectively. Following Kydland and Prescott (1991), the time preference rate is assigned to  $1\%$  (an equivalence of an annual time preference rate of  $4\%$ ). Then we can calculate from (15a) the rate of return to capital as  $r_k=0.0345$ , along the balanced growth path. Set the capital share to the commonly used value  $\alpha=0.36$ , which gives the calibrated capital-real GDP ratio at  $k/y=10.4$  and the calibrated consumption-real GDP ratio at  $c/y=0.745$ , both are very close to the observed value in quarterly data. According to Kendrick (1976), human capital is as large as physical capital. Thus we set the physical to human capital ratio at  $k/h=1$ .

According to Shimer (2005), the monthly separation rate is  $0.034$ , the monthly job finding rate is  $0.45$ , and the elasticity parameter of matching is  $\beta=0.72$ . Therefore, the quarterly separation rate is  $\psi=1-(1-0.034)^3=0.0986$  and the quarterly job finding rate  $\mu=1-(1-0.45)^3=0.834$ . We calibrate the search intensity augmented unemployment measure to  $\mu=s(1-n)=0.065$  and the employment rate can be calibrated to  $0.55$  to match the labor force participation rate  $1-(1-s)(1-n)=n+u=0.615\%$  and the steady-state matching condition  $\mu u = \psi m$ . We then apply the first equality of (12) to set  $v=(1-n)s=0.065$ . Using the

(17a) and (17b), we calibrate  $\eta=B=0.834$ . From (17c), we obtain the value of search intensity:  $s=0.145$ .

While Andolfatto (1996) set the average work time as  $1/3$ , the average figure based on 1965 and 2003 American Time Use Survey for an average man with 13-15 years of schooling is about 28.8%. Thus, we set  $l=0.32$ . Since the observed fraction of time devoted to leisure by the employed is about 60%, we set  $e=1-0.32-0.60=0.08$ , which is consistent with the observed time allocation that an average worker spends 5-10% of time for advanced learning that includes all post-mandatory schooling learning, both on-the-job training and self training). Substituting these into (14b) and (13a), we get  $D=0.0571$  and  $\zeta=0.0020$ . So the exogenous rate of human capital accumulation is at a low rate just about 0.1%. Shimer (2005) normalizes the vacancy-searching worker ratio as  $v/u=1$ , which we follow. Thus, while employed members allocate about 60% of their time ( $1-l-e=0.6$ ) to leisure, the comparable figure for non-employed members is about 85% ( $1-s=0.855$ ).

We then assign a reasonable labor cost of vacancy creation and management as a percentage of employment ( $\Phi/n$ ) at 2.5%. This gives  $\Phi=0.025*0.55=0.0138$ , which can be plugged into (2) to obtain  $A=0.297$ . Since learning effort is non-separable from work effort, we cannot compute directly the labor supply elasticity. But, the learning-augmented labor supply elasticity is given by  $(1/l-1)/\sigma$ . While the labor literature estimates the labor supply elasticity around 0.5, the home production literature gets a higher value at 1.7. We select  $\sigma=1.93$ , which yields a reasonable learning-augmented labor supply elasticity about 1.1. Now, we can use (14d) to calibrate  $\varepsilon=2.229$  and from the definition of  $\Phi$ , we obtain  $\phi=6.073$ .

Next, we use (14c) to compute the BGP value of  $R$  at 2.515. We then apply (16) to calculate  $\gamma_1=1.820$ , which together with the definition of  $R$  implies  $\gamma_2=1.436$ . That is, the employed value their leisure time more than the non-employed, an intuitive result due to the fact that the non-employed may be forced to take leisure involuntarily. Finally, these calibrated parameters can be substituted into (21b) to obtain  $w=0.323$  and  $\Gamma=0.073$ . Thus, the wage discount from its competitive counterpart ( $\bar{w}=0.349$ ), as a consequence of labor-market frictions, is about 7.3%, which seems quite reasonable. See these calibrated values in Table 2.

### Numerical Results

Now, we now simulate the model to examine quantitative effects of two human capital accumulation parameters ( $\zeta$  and  $D$ ) and labor-market parameters ( $B$ ,  $\psi$ , and  $\phi$ ) on endogenous variables of interest. These variables include the balanced growth rate ( $g$ ), effective consumption ( $c/h$ ), physical-human capital ratio ( $k/h$ ), effective output ( $y/h$ ), employment ( $n$ ), unemployment (measured by search intensity augmented job seekers,  $s(1-n)$ ), work effort ( $l$ ), learning effort ( $e$ ), search effort ( $s$ ), workers' job finding rate ( $\mu$ ), firms' employee recruitment rate ( $\eta$ ), and firms' vacancies ( $v$ ). The results are reported in Table 3.

Under the benchmark parameterization, we find that the LD locus is downward-sloping and the LS locus is upward-sloping. While there are many underlying forces driving this outcome, one may identify the dominant forces to gain some intuition. When the job finding rate  $\mu$  is higher, the marginal benefit of employment is lower which

reduces employment  $n$ . Thus, the pseudo labor demand locus is downward-sloping. Turning next to the pseudo labor supply locus, one can see that a higher job finding rate  $\mu$  decreases work effort and increases leisure (cf. (19b)). To restore the equilibrium, investment in human capital is increased, which raises employment (cf. (18)). Thus, the LS locus slopes upward.

In response to human capital accumulation and labor market-improving parameter shifts, there is a large outward shift in the LD locus which outweighs the shift in the LS locus, thus raising both the job finding rate and employment. Moreover, it is noted that in the calibrated equilibrium, Pareto complementarity between employment and work effort in production is a dominant force; as a consequence, the relationship between growth and employment given in the human capital envelope condition, (20a), is always positive. We may thus characterize the growth effects of parameter changes based on their direct effects through (20a) and their indirect effects via the job finding rate and employment in (20a). The numerical results suggest that any shift in human capital-enhancing and labor market-improving parameters always create a negative free-rider effect from thick matching (through  $\mu$ ) and a positive employment creation effect (through  $n$ ): the former reduces growth whereas the latter raises it. All but a shift in  $\zeta$  also generates a positive direct human capital effect through (20a). On balance, each of such shifts affects the growth rate positively. That is, in response to an increase in the experience enhancement parameter, the positive employment creation effect dominates the negative direct human capital effect and the negative free-rider effect from thick matching. In response to other human capital-enhancing and labor market-improving parameter shifts, the positive employment

creation effect and the positive direct human capital effect together dominate the negative free-rider effect from thick matching.

An increase in either the experience enhancement parameter ( $\zeta$ ) or the on-the-job training parameter ( $D$ ) raises learning effort, thus raising employment and economic growth. It is easy to see that the on-the-job training parameter creates stronger employment and growth effects compared to the experience enhancement parameter. Since both parameters raise labor productivity, they also induce labor-market participation and encourage workers to devote greater effort to job search and firms to create more vacancies. Higher search effort raises the unemployment rate, while higher employment lowers it.<sup>4</sup> In the calibrated equilibrium, the search effort effect dominates and hence the unemployment rate is higher in response to an increase in either human-capital enhancing parameter. Due to these offsetting forces, the net increase in unemployment is not as much as the increase in vacancies, thus leading to a higher job finding rate and a lower firm recruitment rate. Moreover, as a result of higher learning and search effort, work effort decreases. Since accumulating human capital is more profitable, there is a factor substitution from physical to human capital. This latter outcome, together with lower work effort and higher vacancy costs, causes the level of effective output to fall, despite a positive growth effect. The fall in effective output subsequently leads to a decrease in effective consumption.

A higher degree of labor-market matching efficacy ( $B$ ), or a smaller separation rate ( $\psi$ ) or a smaller vacancy creation cost ( $\phi$ ) raises employment and job finding rates. While the induced wage incentive effect encourages labor-market participation, learning and search

effort, individual workers may free-ride on the thickness of the labor market that in turns reduces learning and search effort. In the calibrated BGP equilibrium, the wage incentive effect dominates the free-rider effect and, as a result, both output growth and unemployment rates are higher. Moreover, an increase in  $B$  shifts the Beverage Curve outward but a decrease in  $\psi$  and  $\phi$  induce a downward movement along the Beverage Curve in  $(\mu, \eta)$  space. Thus, the former results in higher job finding rate and firm recruitment rate whereas the latter raises job finding rate but reduces firm recruitment rate. While more effective matching or less costly vacancy creation induces more vacancies, a lower separation rate implies that firms retain current employees without the need for creating more vacancies. Similar to the increase in human capital accumulation parameters, these labor-market improvements also cause work effort to fall as a result of higher learning and search effort. For similar arguments, the levels of effective output and effective consumption decrease as well.

Overall, economic growth, employment, labor-market participation, vacancy creation, and learning and search effort are most responsive to changes in the on-the-job training parameter ( $D$ ), followed by job matching and separation rates ( $B$  and  $\psi$ ). The positive growth effect of an increase in the experience enhancement parameter ( $\zeta$ ) is by far the smallest, which is not surprising because of the presence of a negative direct human capital effect. While an increase in the on-the-job training parameter is most effective in fostering growth, it is also associated with the largest decline in work effort, effective output, effective consumption and leisure, as well as the largest increase in the unemployment rate. While job finding rate responds most sensitively to the job matching rate followed by the

on-the-job training parameter, firm recruitment rate responds most sensitively to the on-the-job training parameter followed by the job separation rate and the vacancy creation cost parameter ( $\phi$ ).

Finally, we note that, in response to shifts in any learning and labor-matching parameters, growth and employment always move in the same direction, as do growth and the search intensity augmented unemployment measure ( $u$ ). Thus, if one measures unemployment by purely head counts ( $1-n$ ), there is a negative relationship between growth and unemployment in the long run. If one instead measures unemployment by taking search intensity into account, the long-run relationship between growth and unemployment becomes positive.

### **Human Capital Policy**

Now, we consider two labor-related public policy programs of particular interest:

- (i) an experience enhancement policy enhancing the exogenous component of human capital growth ( $\zeta$ ), which is uniform to all agents (e.g., basic mandatory education);
- (ii) an on-the-job training policy raising the marginal benefit of human capital accumulation ( $D$ ), which favors agents devoting more effort to learning (e.g., on-the-job training and executive learning).

These training programs have been commonly employed in practice and some programs may be intense.<sup>5</sup> We should note that, in order to highlight the role of labor-market frictions, we have abstracted from considering any other imperfections or distortions such as human capital externalities or factor tax distortions. Thus, when the

Hosios rule of efficiency bargaining holds, it is expected that any public policy will not improve upon the decentralized market equilibrium. Nonetheless, one may still compare the growth and welfare effects of the two above-mentioned policies in the revenue-neutral tax-incidence context.

In each of the policy experiments, the subsidy is financed by a lump-sum tax whose value in effective unit is fixed at 1% of the benchmark value of effective output (this effective lump-sum tax turns out to be  $T/h=0.00096$ ). Two preliminary tasks are now in order prior to policy evaluation. First, we must compute the welfare along the BGP. Setting  $h_0=1$ , we can derive the welfare measured by the lifetime utility,

$$\Omega = \frac{1+\rho}{\rho} \left[ \ln\left(\frac{c}{h}\right)^* + \frac{1}{\rho} \ln(1+g) + n\gamma_1 \frac{(1-l-e)^{1-\sigma}}{1-\sigma} + (1-n)\gamma_2 \frac{(1-s)^{1-\sigma}}{1-\sigma} \right] \quad (22a)$$

where we need to modify (6), using (2), (15a), (15b) and the definition of BGP, to derive the after-tax effective consumption:

$$\left(\frac{c}{h}\right)^* = Aq^\alpha (n-\Phi)l - (\delta+g)\frac{k}{h} - \frac{T}{h} \quad (22b)$$

Second, we must compute the relative price of human capital investment in order to compute the rate of subsidy for the two human capital policy experiments. Notice that individual optimization implies that the relative price of human capital investment in unit of outputs ( $P_h$ ) multiplied by the marginal utility of consumption must be equal to the marginal valuation of human capital, which can be used to derive:

$$P_h = MVH' / U_c.$$

We can show this expression is reduced to



$$P_h = \frac{\bar{w}}{D} \quad (23)$$

Table 4 summarizes the results of our key endogenous variables in response to each of the two human capital policies subject to the government budget constraint at a given effective value of lump-sum tax. More specifically, the government budget constraint in each case is given by,<sup>6</sup>

- (i) an experience enhancement policy that increases  $\zeta$  to  $(1+b)\zeta$ :  $b\zeta P_h h = T$ ;
- (ii) an on-the-job training policy that increases  $D$  to  $(1+b)D$ :  $bP_h h D n e = T$ .

Overall, under the benchmark parameterization, an on-the-job training policy is more effective in promoting human capital accumulation and economic growth. In particular, such a subsidy amounted to 1% of effective output evaluated at the benchmark value can raise output growth by 59.3% (which is about 0.267 percentage point increase). This is far more than the effect of an experience enhancement policy (4.3%). Of course, this stronger welfare-enhancing growth effect is accompanied by a larger drop in effective consumption which is welfare-reducing.

However, due to its encouragement for household to participate in the labor market, to seek jobs and to spend time on learning, an on-the-job training policy also generates larger drops in leisure for each of the employed and the unemployed members of the large household ( $1-l-e$  and  $1-s$ ). Since the calibrated value of  $\sigma$  exceeds one, the aggregate value of leisure of the employed ( $n\gamma_1(1-l-e)^{1-\sigma}/(1-\sigma)$ ) is decreasing in  $n$  while the aggregate value of leisure of the unemployed ( $((1-n)\gamma_2(1-s)^{1-\sigma}/(1-\sigma))$ ) is increasing in  $n$ . Thus, the aggregate leisure effect for the employed in response to an on-the-job training policy is

negative, but that for the unemployed is ambiguous. Around the calibrated equilibrium with public policies (see the last column of Table 5), it turns out that the aggregate leisure effect for the unemployed is positive.

We summarize in Table 5 the four components of changes in welfare according to (22a). Changes in welfare come from (i) changes in effective consumption, (ii) changes in the rate of human capital accumulation, (iii) changes in the aggregate leisure effect for the employed and (iv) changes in the aggregate leisure effect for the unemployed. In response to an on-the-job training policy, the negative welfare effect via the aggregate leisure effect for the employed is large, which in conjunction with the negative welfare effect via effective consumption dominates the positive welfare effects via the accumulation of human capital and the aggregate leisure effect for the unemployed. As a result, an on-the-job training policy reduces economic welfare despite its stronger positive effect on the balanced growth rate. For similar arguments, an experience enhancement policy also generates qualitatively similar component effects on welfare, leading to a net reduction in our benchmark economy.<sup>7</sup> The growth-promoting policy instrument by subsidizing human capital discretionarily is associated with a higher welfare cost than subsidizing human capital uniformly.<sup>8</sup>

To highlight the role played by labor-market frictions, we repeat the policy experiments presented in Table 4 in an alternative economy in which such frictions are less severe. We do so by raising the degree of labor-market matching efficacy ( $B$ ) by 5% while maintaining a constant government budget at the value computed from the benchmark economy. The results are summarized in Table 6. By comparing the results with their

counterparts in Table 4, a strong conclusion arrives. That is, as the severity of labor-market frictions diminishes, the effects of these human capital policy programs on key variables all become smaller.<sup>9</sup> Quantitatively, such policy consequences are noticeably smaller even with only a moderate improvement in the job-matching conditions. For example, in this alternative economy with 5% less severe labor-market frictions, the effects of an experience enhancement policy on learning, output growth and employment reduce by about 50% and 20% and 30%, respectively, whereas the comparable figures for an on-the-job training policy are about 40% and 30% and 30%, respectively. Thus, a quantitative evaluation of the effectiveness of public policy in a frictionless Walrasian world is expected to be biased downward severely. This finding is noteworthy because there is a call for reevaluating such human capital policies when the labor market is not frictionless.

### **Conclusion Remarks**

In this paper, we develop an endogenous growth model where sustained human capital accumulation and labor search, matching and entry frictions are integral parts of the economy. Our analysis demonstrates the significant role of labor market frictions in assessing macroeconomic performance and policy effectiveness. We find that an increase in the effectiveness of human capital accumulation or a reduction in the job separation rate or the vacancy creation cost will raise employment, vacancy creation, learning effort and output growth. By conducting two policy experiments that enhance human capital accumulation, we find that an on-the-job training policy is more growth-promoting than an experience enhancement policy, though the on-the-job training policy is also associated

with a higher welfare cost. Our numerical results also suggest that the effects of these public policy programs become larger as the severity of labor-market frictions increase, which reconfirms the important role of labor market frictions.

Our model is subject to several qualifications which call for future research. In the interests of brevity, we mention four possible extensions. First, to simplify the analysis, we assume that the accumulation of human capital only depends on learning effort. It would be interesting to consider the case in which physical capital also contributes to human capital accumulation as modeled by Bond et al. (1996). One may then conduct a full tax-incidence analysis on labor and capital income taxes in the presence of labor-market frictions and compare the results with findings obtained in canonical growth models without frictions. Second, in the present framework, job separation is assumed to be exogenous. It may be extended to allow the separation rate to depend on on-the-job learning effort, as postulated by Mortensen (1988). Such generalization yields an additional margin that may differentiate experience enhancement and on-the-job training policy via endogenous layoff. Third, our framework is ready to be extended to one with credit-market imperfections. The resulting credit constraints may affect human capital investment financing and/or vacancy creation, so the effectiveness of subsidies to learning and vacancy creation need to be reevaluated. Finally, in this study, we focus on the long-run implications of an endogenously growing economy with labor market frictions. Our model may be modified to include technological shocks, as in Merz (1995) and Andolfatto (1996), for quantifying the short-run effects of education and labor-market policies over the business cycle.

The analysis has limitations. First, our results are derived from the assumptions and calibrations of the model. They are relatively strong assumptions and if relaxed, the results might not hold. Next, in the Asia Pacific region, there are many developing countries that have few options for higher education. In the model used in our analysis higher education may be interpreted as vocational schooling. In a revised analysis public subsidy of vocational schooling would be more growth enhancing than subsidizing K-9 education.

Moreover, for ease of analysis, we assume a large household and a large firm. In reality, there are heterogeneous households and heterogeneous firms. If we introduce heterogeneous households, some of the households learn faster than others. While a human capital policy may lead to different rates of human capital accumulation, from the perspective of an economy's overall economic growth point of view, what matters is the average level of human capital. Thus, we think that the relative growth effects of the two human capital implementation policies under study in this paper would remain unchanged.

Finally, following existing literature, our human capital formation is linear. We are aware that linear human capital accumulation does not fit the profiles of wages of low skilled labor.<sup>10</sup> As our paper focuses on overall economic growth, linear human capital accumulation eases the analysis. Nevertheless, this points to an avenue for further research.

There are at least three avenues for further research suggested by the results: to move beyond linear human capital accumulation; to study the effects of increasing unemployment insurance (which would be relatively easy); and on the job training could be incorporated into the model as could heterogeneity in cost of opening vacancies following Fonseca et al. (2001). Heterogeneity is important but it is not a trivial issue

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**Table 1: Benchmark Parameter Values**



time preference rate	$\rho$	0.0100
capital's depreciation rate	$\delta$	0.0200
per capita real economic growth rate	$g$	0.0045
physical capital-human capital ratio	$k/h$	1.0000
fraction of time devoted to work	$l$	0.3200
fraction of time devoted to education	$e$	0.0800
capital's share	$\alpha$	0.3600
labor searcher's share in matching production	$\beta$	0.7200
job separating rate	$\psi$	0.0986
job finding rate	$\mu$	0.8336
labor supply elasticity	$(1/l-1)/\sigma$	1.1000
vacancy creation cost per employment	$\Phi/n$	0.0250
vacancy-searching worker ratio	$v/u$	1.0000
labor force participation rate	$n+u$	0.6150

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**Table 2: Calibration Values**

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capital-output ratio	$k/y$	10.4212
aggregate consumption-aggregate output ratio	$c/y$	0.7447
consumption-human capital ratio	$c/h$	0.0715
coefficient of goods technology	$A$	0.2965
coefficient of matching technology	$B$	0.8336
coefficient of the cost of vacancy creation and management	$\phi$	6.0729
exogenous human capital accumulation rate	$\zeta$	0.0020
maximum rate of endogenous human capital accumulation	$D$	0.0571
rate of return of capital	$r_K$	0.0345
elasticity of substitution of leisure	$\sigma$	1.9318
unemployment measure	$U$	0.0650
fraction of time devoted to employment	$n$	0.5500
search intensity	$s$	0.1445
vacancy creation	$v$	0.0650
coefficient in the utility function	$\gamma_1$	1.8203
coefficient in the utility function	$\gamma_2$	1.4365
cost elasticity of vacancy creation and management	$\varepsilon$	2.2286
employee recruitment rate	$\eta$	0.8336

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**Table 3: Numerical Results**

	$g$	$k/h$	$y/h$	$c/h$	$n$	$l$	$e$
<b>Benchmark</b>	<b>0.004500</b>	<b>1.000000</b>	<b>0.095958</b>	<b>0.071458</b>	<b>0.549969</b>	<b>0.320000</b>	<b>0.080000</b>
$\zeta$ up by 1%	0.005482	-0.001114	-0.000393	-0.000491	0.000344	-0.000319	0.001560
$D$ up by 1%	0.210375	-0.052548	-0.026324	-0.029885	0.055146	-0.060763	0.292090
$B$ up by 1%	0.048535	-0.009987	-0.003365	-0.004524	0.017059	-0.016559	0.068740
$\psi$ up by 1%	-0.047869	0.010005	0.003644	0.004507	-0.016834	0.016904	-0.070124
$\phi$ up by 1%	-0.006256	0.001300	0.000476	0.000588	-0.002200	0.002176	-0.009031

  

	$s$	$\mu$	$\eta$	$\nu$	$u$	$n+u$
<b>Benchmark</b>	<b>0.144503</b>	<b>0.833625</b>	<b>0.833625</b>	<b>0.065031</b>	<b>0.065031</b>	<b>0.615000</b>
$\zeta$ up by 1%	0.000755	0.000009	-0.000022	0.000366	0.000335	0.000343
$D$ up by 1%	0.127404	0.003539	-0.009043	0.064776	0.051425	0.054753
$B$ up by 1%	0.026034	0.012359	0.003960	0.013048	0.004643	0.015746
$\psi$ up by 1%	-0.024810	-0.002265	0.005849	-0.012776	-0.004748	-0.015556
$\phi$ up by 1%	-0.003338	-0.001541	0.003975	-0.006150	-0.000659	-0.002037

Note: Numbers reported in rows 3-7 are percentage changes of key variables from their benchmark values (presented in row 2) due to each exogenous shift.

**Table 4: Policy Experiments: Percentage Changes in Key Variables**

	$b$	$g$	$(y/h)^*$	$(c/h)^*$	$n$	$l$	$e$
<b>Benchmark</b>	NA	<b>0.00450</b>	<b>0.09595</b>	<b>0.071458</b>	<b>0.54996</b>	<b>0.32000</b>	<b>0.080000</b>
		<b>0</b>	<b>8</b>		<b>9</b>	<b>0</b>	
Subsidizing human capital uniformly: $\zeta$	0.079181	0.043300	-0.01309 1	-0.01728 7	0.002684	-0.00248 4	0.012168
Subsidizing human capital discretionarily: $D$	0.032870	0.592740	-0.08548 0	-0.09799 1	0.141204	-0.14936 6	0.749491
	$1-l-e$	$s$	$\mu$	$\eta$	$\nu$	$n+u$	$\Omega$
<b>Benchmark</b>	<b>0.60000</b>	<b>0.144503</b>	<b>0.83362</b>	<b>0.83362</b>	<b>0.065031</b>	<b>0.615000</b>	<b>-476.8560</b>
	<b>0</b>		<b>5</b>	<b>5</b>			<b>06</b>
Subsidizing human capital uniformly: $\zeta$	-0.00029 8	0.005916	0.000068	-0.00017 4	0.002858	0.002677	-0.000271
Subsidizing human capital discretionarily: $D$	-0.02027 0	0.363742	0.011336	-0.02857 0	0.174768	0.139852	-0.004683

Note: Variables  $(c/h)^*$  and  $(y/h)^*$  represent after-tax effective consumption and output, respectively; see also Table 2.

**Table 5: Policy Experiments: Decomposition of Changes in Welfare**

Welfare Decomposition	$\Omega$	(1) Human Capital Growth	(2) Effective Consumption	(3) Leisure of the Employed	(4) Leisure of the Unemployed
Subsidizing human capital uniformly: $\zeta$	-0.000271	0.004108	-0.003693	-0.001085	0.000400
Subsidizing human capital discretionarily: $D$	-0.004683	0.056167	-0.021843	-0.059773	0.020767

Note: See Table 2.

**Table 6: Policy Experiments for an Alternative Economy: Percentage Changes in Key Variables**

	$b$	$g$	$(y/h)^*$	$(c/h)^*$	$n$	$l$	$e$
<b>Equilibrium (B=0.8336*1.05)</b>	NA	0.005488	0.094406	0.070034	0.592388	0.297378	0.103502
Subsidizing human capital uniformly: $\zeta$	0.080544	0.034317	-0.013046	-0.017286	0.001815	-0.001625	0.006221
Subsidizing human capital discretionarily: $D$	0.030040	0.399985	-0.074664	-0.085479	0.099939	-0.115446	0.436289
	$1-l-e$	$s$	$\mu$	$\eta$	$v$	$\Omega$	
<b>Equilibrium (B=0.8336*1.05)</b>	<b>0.59911</b>	<b>0.161794</b>	<b>0.885418</b>	<b>0.849831</b>	<b>0.068711</b>	<b>-476.457493</b>	<b>9</b>
Subsidizing human capital uniformly: $\zeta$	-0.000268	0.004398	0.000067	-0.000172	0.001987	-0.000253	
Subsidizing human capital discretionarily: $D$	-0.018069	0.273534	0.010452	-0.026382	0.129744	-0.003533	

Note: Numbers reported in rows 3-4 are percentage changes of key variables from their equilibrium values with B=0.8336\*1.05 (presented in row 2) due to each educational subsidy.

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## Endnotes

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<sup>1</sup> See, for example, Shimer (2005).

<sup>2</sup> See Becker (1962) and Pencavel (1972) for a discussion on general versus job-specific training and Werther et al. (1995) for issues concerning executive learning.

<sup>3</sup> It is also difficult to prove analytically the existence of a balanced growth path with positive growth, though our calibration exercises ensure such a property.

<sup>4</sup> The positive effect of training on job search intensity is consistent with empirical findings in Barron, Black and Loewenstein (1989).

<sup>5</sup> For example, in terms of firm training alone, Barron, Black and Loewenstein (1989) document that it takes about 29% of the total employment hours for new hires over the first three months since the start of the jobs.

<sup>6</sup> The required rates of subsidy for the two experiments are about 7.9% and 3.3%, respectively. Notice that these policies can be evaluated based only on the relative price of human capital investment.

<sup>7</sup> Recall that the dynamic search equilibrium features efficient wage bargaining. In the absence of preference/ production externalities, distortionary taxes, or other imperfections, education and investment subsidies are not expected to improve welfare. Should one include uncompensated human capital spillovers (cf. Lucas 1988) or factor income taxation (cf. Bond et al. 1996), these subsidy programs may become welfare-enhancing. Thus, our discussion here only focuses on relative welfare comparisons between different policies, rather than the absolute welfare gains/losses associated with each policy.

<sup>8</sup> See also Miner (1962) on the costs of training which may be one of the reasons that explain why the negative welfare effects show up here.

<sup>9</sup> This conclusion applies to all individual macroeconomic variables. Here, we exclude the welfare measure because it is an aggregator of several macroeconomic variables.

<sup>10</sup> See Acemoglu and Autor (2012) for discussion about the interaction between human capital and technology.