Games Suppliers and Producers Play: Upstream and Downstream Moral Hazard with Unverifiable Input Quality

Brishti Guha
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Brishti Guha
Singapore Management University

Abstract

We pin down the optimal relational contract between an input supplier and a final goods producer given a framework of bilateral moral hazard with variable but non-verifiable input quality. Given the inability of third parties to verify input quality, each party has an incentive to cheat the other by making a false claim about input quality. We derive the contract which (a) induces honest behavior and brings about the Pareto superior “first-best” outcome for the widest possible range of exogenous parameters, and (b) maximizes the Nash product of both parties’ payoffs subject to incentive compatibility. An interesting feature of the optimal contract is that it is of a “fixed-price” variety with the final producer paying the supplier the same transfer price whether he has been supplied a high or low quality input when the agreement was to supply high quality. This contrasts with the traditional incomplete contracting literature where fixed-price contracts (e.g., payment of a fixed wage to workers) was optimal only in the full information case – while ours is a case of incomplete information. The contrast is rooted both in the bilateral nature of the moral hazard we consider and in the repeated game framework we use. We also pinpoint the exact transfer price in the optimal contract, which may vary for different parameter ranges, and show how the best contract differs from the optimal contract under complete contracting.

Keywords: Incomplete contracting, upstream and downstream moral hazard, repeated games, Nash bargaining.

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1 School of Economics and Social Sciences, Singapore Management University, 90 Stamford Road, Singapore 178903. E-mail: bguha@smu.edu.sg. The bulk of this work was done while I was a PhD student at Princeton University. I am grateful to Avinash Dixit for valuable input.
1. Introduction

In this paper we apply an incomplete contracting framework to an analysis of bilateral moral hazard between a supplier of inputs and a final producer who uses the input for the production of a final good. The input is of variable quality but input quality is non-verifiable. This introduces scope for cheating. In the absence of any non-verifiability, quality-contingent transfer prices would simply be determined by a process of Nash bargaining. However, because of the non-verifiable aspect of input quality, these prices would not be fully enforceable. Either party may have an incentive to make a false claim about input quality. We characterize the optimal relational contract, where optimality is defined in terms of satisfying incentive compatibility for the greatest range of parameters, and of maximizing surplus subject to incentive compatibility. Our analysis delivers some surprising results which contrast with the traditional literature.

An interesting feature of the optimal contract we derive is that it is of a “fixed-price” variety with the final producer paying the supplier the same transfer price whether he has been supplied a high or low quality input when the agreement was to supply high quality. This contrasts with the traditional incomplete contracting literature where fixed-price contracts (e.g., payment of a fixed wage to workers) was optimal only in the full information case – while ours is a case of incomplete information. The contrast is rooted both in the bilateral nature of the moral hazard we consider and in the repeated game framework we use. We also pinpoint the exact transfer price in the optimal contract, which may vary for different parameter ranges, and show how the best contract differs from the optimal contract under complete contracting.

Our problem is related to the general body of literature investigating the achievement of efficient economic outcomes given non-verifiabilities that tend to create moral hazard. Such literature includes papers by Greif (1991, 1996), Ellickson (1992), and Dixit (2003). From a contract theory angle, incentive
compatibility and moral hazard have of course been studied from much earlier – going back to Holmstrom and Milgrom (1982) and Myerson (1981). Our work also relates to a rich literature on hold-up problems, dating back to Oliver Williamson (1975, 1979) and Grossman and Hart (1986). This literature deals with variants of a situation where a “seller” is discouraged from transacting or from making a socially optimal investment for fear that the fruits of his efforts will be appropriated by a “buyer”, or that he will be cheated out of his fair share. This would lead to a Pareto inferior outcome that in extreme cases corresponds to the disappearance of the market. The seller’s fears, in turn, are rooted in the incomplete nature of the contract. More recent literature on hold-up problems includes Gul (2001). Gul analyzes a one-buyer, one-seller case where the seller has all the bargaining power, and shows how the hold-up problem could be resolved if (a) the buyer’s investment is unobservable by the seller, and (b) the seller makes frequently repeated offers. Other literature that indirectly analyzes hold-up problems includes work on vertical integration versus outsourcing. Since final producers who outsource the initial stage of their production processes to independent contractors (suppliers) are able to maintain less control relative to those who integrate, moral hazard and non-verifiability play a definite role in the decision whether to outsource or integrate.

While the final producer could integrate vertically with the input supplier in response to the severity of the hold-up problem, it is possible that the rising costs of control as the size and scope of the enterprise increase may make integration an expensive proposition. In addition, the economics of vertical integration have been thoroughly explored. We pursue an alternative route, analyzing the conditions needed to bring about the first-best outcome in a simple indefinitely repeated game between the supplier and the producer. We use a Nash bargaining framework, implicitly assuming that both parties to the transaction have equal bargaining power. As the total surplus from production of a high quality good (which requires a high quality input) is larger than the corresponding

surplus from low quality good production, the first best outcome is that the supplier supply an input of high quality and this be used to make a high quality good. Moreover, neither party should have an incentive to cheat. We determine the transfer prices which would ensure this for different ranges of parameters, and find that the *optimal* transfer prices may be different from those determined by unrestricted Nash bargaining\(^3\). Essentially, in this game the low quality outcome serves as a threat point for the high quality outcome, while the threat point for the low quality outcome in turn is the disappearance of the market, or no transaction.

In section 2 of this paper we analyze moral hazard from the perspective of both the input supplier and the final producer, deriving the optimal relational contract for different ranges of exogenous parameters. We define an optimal relational contract as one that (a) would lead to the first-best outcome obtainable under complete contracting, under the weakest possible restrictions, and (b) would maximize the Nash product, subject to incentive compatibility. We focus on a simple model with only one input supplier and one final producer. In section 3 we discuss the implications of some of the features of our optimal contract. Section 4 concludes.

\section*{2. Moral Hazard and the Optimal Contract}

We consider an input supplier who can produce an input of either high or low quality, incurring a cost \(c_H\) for a high-quality and a cost \(c_L\) for a low quality input \((c_H>c_L)\). Costs are exogenous constants, implying an infinitely elastic supply of factors of production to the supplier and constant returns to scale in the production of the input.

\(^3\) By unrestricted Nash bargaining, we mean a Nash bargaining exercise not subject to any incentive constraints (which spring from moral hazard).
The other player in the game is a final goods producer who buys the input from the supplier for a transfer price. He can costlessly transform one unit of the high quality intermediate into one unit of a high quality product and a unit of the low quality input into one of a low quality good.

Both players have zero opportunity cost and find it optimal accordingly to continue in their relationship for any non-negative payoff. The relationship is one-to-one and exclusive: neither agent can engage more than one partner at a time. This implies that possible complications arising from competition between rival producers and/or suppliers can be ignored.

Final product prices are set exogenously at $V_H$ and $V_L$ respectively for high and low quality goods ($V_H > V_L$). This would be the case in a small open economy facing fixed world prices. We also assume that the total surplus from high quality production exceeds that from low quality production, so that a division of this surplus is conceivable that leaves both supplier and producer better off. Thus we have

$$V_H - c_H > V_L - c_L$$

or

$$V_H - V_L > c_H - c_L \quad (1)$$

If contracts were complete – which would be the case if input quality were verifiable – any Pareto-optimal contract would specify high-quality supply and a transfer price $p_H'$ for the high-quality input. Given the fact of high-quality production, Nash bargaining would set $p_H'$ at the value that maximizes $(V_H - p_H')(p_H' - c_H)$. Thus,

$$p_H' = \frac{V_H + c_H}{2} \quad (2)$$

Any possibilities of cheating would have been ruled out by complete contracting, so there would be no scope for either the supplier or the producer to cheat by making false claims about input quality.
However, we analyze a situation where input quality is non-verifiable, with indefinitely repeated interactions between the input supplier and the final goods producer. Incomplete contracting implies that the understanding between the supplier and the producer does not represent a fully enforceable contract. So we need to take into account both parties’ incentives to cheat. The producer has an incentive to cheat if, by falsely claiming that the input he was supplied is of inferior quality, he can get away with paying the supplier an amount he would be paid for a low quality input. The supplier has an incentive to cheat if he can claim that the input is of high quality, but in reality supplies a low quality input, saving on his costs (making a one-time cheating gain of $c_h - c_l$). Such cheating is possible because input quality is non-verifiable. In a one-shot game, incentives to cheat would prevail, as no penalty is borne by the cheat—therefore, low quality is the Nash equilibrium in the stage game. In the infinitely repeated game that we examine, however, cheating can be punished by reversion to the Nash equilibrium of the one shot game—the low quality outcome. The division of surplus in the low quality outcome, in turn, would be driven by the threat point of transactions breaking down, in which case each party would get zero.

In this environment of incomplete contracting, a “relational contract” specifies transfer prices $p_h$, the price which is to be paid if the producer declares that he has been supplied high quality, and $p_L$, the price which is to be paid if the producer declares that he has been supplied low quality. Unlike input quality, quantity delivered and prices are verifiable, so either $p_h$ or $p_L$ will have to be paid. Both parties are opportunistic and will cheat if it is profitable for them to do so. However we assume that when honesty and cheating are equally profitable alternatives, they will choose to be honest.

Feasible strategies open to the two players are as follows. At the beginning of each period, they agree on whether high quality or low quality is to
be supplied. If the agreement is to supply low quality, the supplier supplies a low quality input and the producer pays him a Nash-bargaining determined transfer price $p_L'$ [which is determined with reference to the threat point of no transaction, with both parties getting zero]. If the agreement is to supply high quality, the supplier can then either supply a high quality input, or cheat by actually supplying low quality. If the supplier supplies low quality when the agreement was to supply high quality, the producer pays him $p_L$ as specified in the relational contract. If the supplier supplies high quality, the producer can either pay him $p_H$, or cheat by paying him $p_L$, taking advantage of the non-verifiability of input quality. Depending on the actual quality of the input supplied, the producer produces either a high-quality or a low-quality good, and payoffs are realized. The game is then repeated indefinitely.

Our objective is to determine the transfer prices specified by the optimal relational contract. We regard a relational contract as optimal if (a) it eliminates incentives to cheat for the widest possible range of exogenous parameters, and (b) if it maximizes the Nash product subject to (a). The optimal contract may however differ (as we show below) depending on parameters and will also differ (with regard to at least one transfer price, and sometimes both) from the transfer prices determined by unrestricted Nash bargaining under complete contracting.

Now the transfer price in the low-quality bargain, $p_L'$, is determined through Nash bargaining, driven by the fact that, without an agreement on it, the two players would not even be able to capture the surplus on low quality. Then $p_L'$ maximizes the Nash product $(V_L - p_L)(p_L - c_L)$ implying

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4 We not only discuss the transfer prices but also their relationship with the question of when payment should be made – whether it should be made upfront, in response to the supplier’s claim about input quality, or whether it should be made later, after the producer has obtained the input and had a chance to observe its true quality.
\[ p_L = \frac{V_L + c_L}{2} \]  

(3)

In the low quality bargain, therefore, each party ends up with a surplus of \( \frac{V_L - c_L}{2} \). The low quality outcome, in turn, serves as the threat point for bargaining on the high quality outcome. With incomplete contracting, however, bargaining is subject to incentive constraints ruling out cheating for both the producer and the supplier. Therefore, the objective is to find a relational contract specifying \( p_H \) and \( p_L \) which maximize

\[
\phi(p_H) = [V_H - p_H - \frac{V_L - c_L}{2}][p_H - c_H - \frac{V_L - c_L}{2}] 
\]

subject to

\[
r(p_H - p_L) \leq V_H - p_H - \frac{V_L - c_L}{2} \quad (5)
\]

and

\[
r(c_H - c_L) \leq p_H - c_H - \frac{V_L - c_L}{2} \quad (6)
\]

While (4) expresses the Nash product using the low quality outcome as a threat point, (5) shows the producer’s incentive constraint and (6) the supplier’s. The producer has to weigh his one time cheating gains\(^5\) against the prospect of receiving only low quality inputs (instead of high quality) ever after. For convenience of notation we express this condition in terms of interest rates \( r \) rather than in terms of the discount factor \( \delta \) (\( \delta = \frac{1}{1+r} \)). Similarly the supplier has to weigh his one-time saving on input costs as a result of supplying low quality instead of high, against the punishment of reversion to low quality ever after.

\(^5\) The producer’s one time cheating gains amount to \( p_H - p_L \) as he can avoid paying this amount by falsely asserting that the input was of low quality.
For the moment, suppose we are in a complete contracting environment so that (4) is maximized without the constraints (5) and (6). Our objective in considering this possibility is to examine under what restrictions the “first-best” or “complete contracting” transfer prices would satisfy incentive constraints and rule out cheating. We will then proceed to show that a different contract can do better in the sense of being able to rule out cheating under weaker conditions.

Unrestricted Nash bargaining would lead to the transfer price

$$p_H^* = \frac{V_H + c_H}{2}$$

Now when would the pair of transfer prices \((p_H^*, p_L^*)\) (given by (2) and (3)) satisfy the incentive constraints (5) and (6)? To answer this question we substitute in the values for \(p_H^*\) and \(p_L^*\) for \(p_H\) and \(p_L\) in (5) and (6). Then comparing the left hand sides of (5) and (6), we see that the left hand side of (6) is smaller, because

$$p_H^* - p_L^* = \frac{(V_H - V_L) + (c_H - c_L)}{2} > \frac{(c_H - c_L) + (c_H - c_L)}{2} \quad \text{[from (1)]}$$

Thus we have

$$c_H - c_L < p_H^* - p_L^*$$

So (5) is the binding constraint. After simplification we see that (5), and therefore (6), are satisfied for

$$c_H - c_L < \frac{1-r}{1+r} (V_H - V_L) \quad \text{(7)}$$

Thus the “complete contracting” transfer prices will be incentive compatible subject to (7). We now consider whether any other set of transfer prices specified by the relational contract could do better – no longer restricting them to be equal to the complete contracting transfer prices. From (5), we see that the producer’s incentive to cheat can actually be eliminated by fixing \(p_L = p_H\). This would imply that irrespective of whether the producer claims he
has been supplied low quality instead of high quality, he has to pay the same price this period. If indeed the supplier had supplied low quality, the producer can punish him by reverting to the low quality outcome in all subsequent periods – but in the current period, no punishment is inflicted. This eliminates the producer’s incentive to cheat by making it impossible for him to make any gain by a false assertion of having received a low quality input. Substituting \( p_L = p_H \) in the incentive constraints (5) and (6), we get

\[
p_H \leq V_H - \frac{V_L - c_L}{2}
\]

(8)

And

\[
p_H \geq c_H + r(c_H - c_L) + \frac{V_L - c_L}{2}
\]

(9)

(8) is not a binding constraint (as is obvious from (1)). For the existence of a \( p_H \) satisfying both (8) and (9), we need

\[
V_H - \frac{V_L - c_L}{2} > c_H + r(c_H - c_L) + \frac{V_L - c_L}{2}
\]

which simplifies to

\[
V_H - V_L > (1 + r)(c_H - c_L)
\]

(10)

Now a comparison of (7) and (10) tells us that (10) is a weaker condition than (7). Thus it is possible to find an incentive compatible relational contract for a greater range of parameters if we look at the class of contracts with \( p_L = p_H \), than if we restrict ourselves to a relational contract specifying transfer prices identical to those obtained under complete contracting.

Next we turn our attention to characterizing the optimal contract among the class of contracts with \( p_L = p_H \). To do this we first calculate the derivative of the maximand in (4). This turns out to be

\[
\phi'(p_H) = V_H + c_H - 2p_H
\]

We denote the upper limit on \( p_H \) implied by (8) by \( p_H^{(\text{max})} \) and the lower limit implied by (9) by \( p_H^{(\text{min})} \), our objective being to find the optimal \( p_H \). We now
evaluate the derivative of the Nash product at the end points of the range 
\((p_H(\text{min}), p_H(\text{max}))\) – which is the range in which any incentive-compatible contract in the class \(p_L = p_H\) has to lie. We find that

\[
\phi'(p_H(\text{max})) = V_H + c_H - 2[V_H - \frac{V_L - c_L}{2}] = (V_L - c_L) - (V_H - c_H) < 0
\]

Now

\[
\phi'(p_H(\text{min})) = V_H + c_H - 2[c_H + r(c_H - c_L) + \frac{V_L - c_L}{2}]
\]

or

\[
\phi'(p_H(\text{min})) = (V_H - V_L) - (1 + 2r)(c_H - c_L)
\]

This may be positive or negative depending on parameter values. First consider the subset of parameters where this derivative is positive, that is, where

\[
V_H - V_L > (1 + 2r)(c_H - c_L)
\]

(11)

We note that (11) being a stronger condition than (10), the interval \((p_H(\text{min}), p_H(\text{max}))\) is always non-empty for the range of parameters satisfying (11). Now \(\phi(p_H)\) being a concave function – we can easily check that the second derivative is \(-2 < 0\) – if its derivative is positive at the lower end point and negative at the higher, it must have an interior maximum. This occurs where

\[
\phi'(p_H) = V_H + c_H - 2p_H = 0
\]

or

\[
p_H = \frac{V_H + c_H}{2} = p_H'
\]

Thus the optimal transfer price for this range of parameters is equal to the transfer price for the high quality input under complete contracting. The difference is that the relational contract specifies that this transfer price be paid even if the producer were to claim that he has received low quality instead of high (if the complete contracting outcome were to be mimicked exactly, this would entail the producer paying out \(p_L'\) following a claim of having received low quality). Therefore, for the range of parameters satisfying (11) [and therefore, automatically (10)], the optimal relational contract is
\[ p_L = p_H = \frac{V_H + c_H}{2} = p_H' \]  

(12)

What about the range of parameters where (11) is not satisfied, though (10) is? For this parameter range, although there is a non-empty interval of incentive-compatible contracts in the class \( p_L = p_H \), the derivative of the Nash product at the lower (and not just the upper) end point is negative. This is the range where

\[ (1 + 2r)(c_H - c_L) > V_H - V_L > (1 + r)(c_H - c_L) \]  

(13)

so that \( \phi'(p_H(\text{min})) < 0 \). In this case it is optimal to set \( p_H = p_H(\text{min}) \). This means that it is optimal then to set \( p_H \) barely high enough to satisfy the supplier’s incentive constraint, for this range of parameters. So the optimal relational contract in the range (13) is

\[ p_L = p_H = c_H + r(c_H - c_L) + \frac{V_L - c_L}{2} = p_H(\text{min}) \]  

(14)

We note that contracts (12) and (14) are superior to the mere adoption of the complete contracting transfer prices in an incomplete contracting environment, because they succeed in removing incentives to cheat over a greater range of parameters. While contract (12), applicable to parameter ranges satisfying (11), has one element in common with the unrestricted contract, contract (14), applicable to parameters satisfying (13), has none. With regard to timing, for relational contracts of the sort captured by (12) or (14), it does not matter whether payment for the input is made up-front or after the producer has taken possession of the input. In the latter case, the producer will not have an incentive to cheat by making a false assertion about input quality, as this does not gain him anything (\( p_L = p_H \)). For an up-front payment, the producer may in addition not even have the chance to lie about input quality, as he may not have the opportunity to observe this before making his payment. In either case, the
supplier’s incentive to provide low quality where high quality has been agreed upon is ruled out by keeping $p_H$ sufficiently high.

With reference to condition (10), we note that the ability to sustain the high-quality outcome through relational contracting of the sort we have analyzed is larger, the larger the quality difference in product market prices relative to the quality difference in supplier’s costs, and the smaller $r$ is. The analysis of this section thus leads us to the following proposition:

**Proposition 1**: Subject to condition (10), that is

$$V_H - V_L > (1 + r)(c_H - c_L)$$

(10)

it is possible to sustain a relational contract which ensures that along the equilibrium line of play, high quality is supplied and neither the supplier nor the producer has any incentive to cheat. The optimal relational contract is given by (12) for parameters satisfying (11), and by (14) for parameters satisfying (13). The off-equilibrium threats involve a reversion to the low quality outcome in all future periods.

### 3. Discussion

The optimal contract we derive has some features which distinguish it from other incomplete information contracts analyzed in the literature. In particular, it is of the fixed price variety in the sense that the producer pays the supplier the same input price regardless of whether high or low quality has been delivered, provided the agreement was to supply high quality. If we recall early work on moral hazard, such “fixed-price” contracts were only optimal in cases of full information – not in the case of incomplete information. For example, take the case of a manager who has to decide how to pay a worker whose effort choice is

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6 Since our entire analysis in this section has been leading up to this proposition, we offer no further proof.
unobservable. The analysis of this case discusses why fixed-wage contracts are definitely not optimal given the unobservability of effort choice. Why do our findings differ?

There are two reasons underlying these differences. First, we consider bilateral moral hazard. It is true that if only the supplier had an incentive to cheat, paying him a fixed price may have been detrimental to incentives – particularly in a one-period interaction. However, we also consider the possibility that the producer may cheat by lying about whether high quality was supplied.\(^7\) This incentive to lie is eliminated by the “fixed price” feature of our contract. We then use the supplier’s incentive compatibility problem, as well as the objective of maximizing both parties’ Nash product, to pin down the exact price. The second reason why our results are different is that we use a repeated game framework. Therefore, the fact that the supplier is not punished for cheating this period (that is, for supplying low quality when high quality had been agreed upon) does not imply that he can escape scot-free. In fact reversion to the one-stage “low-quality” Nash equilibrium in all future periods constitutes the off-equilibrium threat sustaining our equilibrium.

We also observe that if the surplus of the high quality outcome over the low quality outcome is relatively small – so that, say, (13) may hold but not (11) – then it is optimal for producers to pay suppliers a transfer price that is lower than what would be paid under full information. If the surplus were high enough for (11) to hold, then the optimal amount to be paid is equal to the full-information amount, and is to be paid whether or not the producer claims he has been supplied low instead of high quality.

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\(^7\) Thus we take into account the possibility that the second party to a transaction may cheat by lying about having been cheated by the first party.
4. Summary and Conclusions

In this paper we analyzed a case of sustaining “order without law” where an input supplier sells an input of non-verifiable quality to a final goods producer. With complete contracting, Nash bargaining would be used to agree upon quality-contingent transfer prices. In reality however the inability of third parties to verify input quality implies that we need to take into account both parties’ incentives to cheat by making false assertions about input quality. We examine the optimal relational contract – one which makes it possible to sustain the Pareto superior high quality outcome for the widest range of exogenous parameters, and also maximizes the Nash product subject to incentive compatibility. The optimal contract always has the feature that once high quality has been agreed upon, the producer has to pay the same price regardless of whether or not he claims he has been supplied low quality instead. The exact transfer price it is optimal to pay will differ for different parameter ranges, corresponding in one case to an interior solution and in the other to a corner solution. Throughout the off-equilibrium punishment considered is reversion to the low quality Nash equilibrium of the one-shot game. We then briefly highlight some features of our optimal contract, including its differences from the cases of moral hazard traditionally analyzed in the literature.

As regards the debate on vertical integration versus outsourcing, one motive for vertical integration is to control possible opportunism by the other party. To the extent that “relational contracting” of the sort analyzed in this chapter suffices to promote first best outcomes, this particular incentive for integration would disappear. An interesting result is that the parameters entering conditions (10), (11) and (13) being common knowledge, it is easy for both parties to determine whether the conditions for successful relational contracting
indeed hold, and if so, what would constitute the optimal relational contract. This could be a key factor in their decision to outsource or integrate.

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